

# Physical Theories, Eternal Inflation, and Quantum Universe

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## Abstract

Infinites in eternal inflation have long been plaguing cosmology, making any predictions highly sensitive to how they are regulated. The problem exists already at the level of semi-classical general relativity, and has a priori nothing to do with quantum gravity. On the other hand, we know that certain problems in semi-classical gravity, for example physics of black holes and their evaporation, have led to understanding of surprising, quantum natures of spacetime and gravity, such as the holographic principle and horizon complementarity.

In this paper, we present a framework in which well-defined predictions are obtained in an eternally inflating multiverse, based on the principles of quantum mechanics. We show that the *entire* multiverse is described *purely* from the viewpoint of a single “observer,” who describes the world as a quantum state defined on his/her past light cones bounded by the (stretched) apparent horizons. We find that quantum mechanics plays an essential role in regulating infinities. The framework is “gauge invariant,” i.e. predictions do not depend on how spacetime is parametrized, as it should be in a theory of quantum gravity.

Our framework provides a fully unified treatment of quantum measurement processes and the multiverse. We conclude that the eternally inflating multiverse and many worlds in quantum mechanics are the same. Other important implications include: global spacetime can be viewed as a derived concept; the multiverse is a transient phenomenon during the world relaxing into a supersymmetric Minkowski state. We also present a theory of “initial conditions” for the multiverse. By extrapolating our framework to the extreme, we arrive at a picture that the entire multiverse is a fluctuation in the stationary, fractal “mega-multiverse,” in which an infinite sequence of multiverse productions occurs.

The framework discussed here does not suffer from problems/paradoxes plaguing other measures proposed earlier, such as the youngness paradox, the Boltzmann brain problem, and a peculiar “end” of time.

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# 1 Introduction and Summary

A combination of eternal inflation [1] and the string landscape [2] provides a theoretically well-motivated framework in which we might understand how nature selects our local universe to take the form as we see today. On one hand, the string landscape provides an enormous number of quantum field theories, of  $O(10^{500})$  or more, as consistent vacuum solutions to string theory. On the other hand, eternal inflation physically realizes all these vacua in spacetime, allowing us to have an anthropic understanding for the structure of physical theories [3], including the value of the cosmological constant [4].

This elegant picture, however, suffers from the issue of predictivity arising from infinities associated with eternally inflating spacetime, known as the measure problem [5]. In fact, it is often said that *“In an eternally inflating universe, anything that can happen will happen; in fact, it will happen an infinite number of times”* [6]. While this sentence captures well the origin of the issue, its precise meaning is not as clear as one might naively think. Consider the part “anything that can happen will happen.” One might think it to mean that for a given initial state, anything that is allowed to happen in the landscape will happen. However, any single “observer” (geodesic) traveling in eternally inflating spacetime will see only a finite number of vacua before he/she hits a (big crunch or black hole) singularity or is dissipated into time-like infinity of Minkowski space. Namely, for this observer, something that can happen may not happen. We are thus led to interpret the phrase to mean either “anything that can happen will happen with a nonzero probability” or “anything that can happen will happen at least to somebody.” The former interpretation, however, leads to the question of what probability we are talking about, and the latter the question of whether considering the histories of *all* different observers physically makes sense despite the fact that they can never communicate with each other at late times. Similar questions also apply to the part “it will happen an infinite number of times.” Indeed, any randomly selected observer will see a particular event only a finite number of times.

The first aim of this paper is to present a view on these and related issues in eternal inflation, by providing a well-defined framework for prediction in eternally inflating spacetime, i.e. in the eternally inflating multiverse. Our framework is based on the following three basic observations:

- In any physical theory, predicting (or postdicting) something means that, given what we “already know,” we obtain information about something we do *not* know. In fact, there are two aspects in this: one is the issue of dynamical evolution and the other about probabilities. Suppose one wants to predict the future (or explore the past) in a classical world, given perfect knowledge about the present. This requires one to know the underlying, dynamical evolution law, but only that. On the other hand, if one’s future is determined only probabilistically, or if one’s current knowledge is imperfect, then making predictions/postdictions requires a

definition of the probabilities, *in addition to* knowing the evolution law. The measure problem in eternal inflation is of this second kind.

- Let us assume that the underlying evolution law is known (e.g. the landscape scalar potential in string theory). Making predictions/postdictions is then *equivalent* to providing relative weights among all physical possibilities that are consistent with the information we already have. In particular, since our knowledge is in principle limited to what occurred within our past light cone, having a framework for prediction/postdiction is the same as coming up with a prescription for finding appropriate *samples for past light cones* that are consistent with our prior knowledge.
- From the point of view of a single observer, infinities in eternal inflation arise only if one repeats “experiments” an infinite number of times, i.e. only if the observer travels through the multiverse an infinite number of times. This implies that by considering a huge, but finite, number of observers emigrating from a fixed initial region at a very early moment, only a finite number of—indeed, only a very small fraction of—observers see past light cones that are consistent with the conditions we specify as our prior knowledge. In particular, if our knowledge contains exact information about any non-static observable, then the number of past light cones satisfying the prior conditions *and* encountered by one of the observers is always finite, no matter how large the number of observers we consider.

These observations naturally lead us to introduce the following general framework for making predictions, which consists of two elements:

- (i) We first phrase a physical question in the form: given a set of conditions  $A$  imposed on a past light cone, what is the probability of this light cone to have a property  $B$  or to evolve into another past light cone having a property  $C$ ? We will argue that any physical questions associated with prediction/postdiction can always be formulated in this manner.
- (ii) We then find an ensemble of past light cones satisfying conditions  $A$  by “scanning” what observers *actually see*. Specifically, we consider a set of geodesics emanating from randomly distributed spacetime points on a fixed, “initial” (space-like or null) hypersurface at a very early moment in the history of spacetime. We then keep track of past light cones along each of these geodesics, and if we find a light cone that satisfies  $A$ , then we “record” it as an element of the ensemble. Answers to any physical questions can then be obtained by simply counting the numbers of relevant light cones:

$$\frac{\mathcal{N}_{A \cap B}}{\mathcal{N}_A} \rightarrow P(B|A), \quad \frac{\mathcal{N}_{A \rightarrow C}}{\mathcal{N}_A} \rightarrow P(C|A), \quad (1)$$

where  $\mathcal{N}_A$ ,  $\mathcal{N}_{A \cap B}$ , and  $\mathcal{N}_{A \rightarrow C}$  are the numbers of the recorded light cones that satisfy the specified conditions. Note that these numbers are countable and finite if conditions  $A$  involve

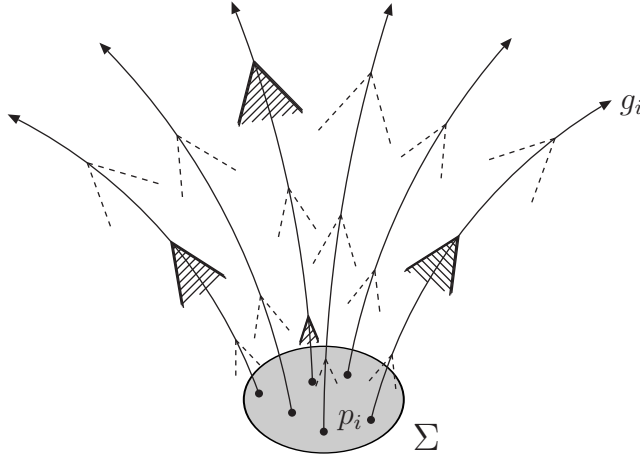


Figure 1: A schematic picture for obtaining samples of past light cones that satisfy specified conditions  $A$ . The light cones to be selected are depicted by shaded triangles (with the figure showing  $\mathcal{N}_A = 4$ ). Note that a single geodesic may encounter relevant light cones multiple times in its history.

a specification of any non-static observable, e.g. the value of a parameter that can play the role of time. The predictions in Eq. (1) can be made arbitrary precise by making the number of the geodesics arbitrarily large.

A schematic depiction of the procedure of sampling past light cones is given in Fig. 1. Note that this procedure corresponds precisely to simulating the entire multiverse many times as viewed from a single observer (geodesic). Also, in this method, any time coordinate becomes simply a spurious parameter, exactly like a variable  $t$  used in a parametric representation of a curve on a plane,  $(x(t), y(t))$ . It plays *no role* in defining probabilities, and time evolution of a physical quantity  $X$  is nothing more than a correlation between  $X$  and a quantity that can play the role of time, such as the average temperature of the cosmic microwave background (CMB) in our universe.

It is illuminating to highlight a close similarity between the prescription here and the formulation of usual quantum mechanics. Consider a quantum mechanical system in a state  $|\psi\rangle = \sum_i a_i |\phi_i\rangle$  with  $a_i \neq 0$ . If we perform an experiment many times on this system, we would find that anything that can happen (represented by  $|\phi_i\rangle$ ) will happen (with a nonzero probability  $|a_i|^2 / \sum_j |a_j|^2$ ). Moreover, by performing it an infinite number of times, all these results occur an infinite number of times. In our prescription,  $|\psi\rangle$  corresponds to the multiverse, while  $|\phi_i\rangle$  to a collection of past light cones along a geodesic in this spacetime. Performing an experiment then corresponds to sending an observer (geodesic) from an initial hypersurface located at the “beginning” of spacetime.

The second aim of this paper is to formulate the above framework, described using the semi-classical picture, in the context of quantum mechanics, where we will see that the similarity just illustrated is more than an analogy. The necessity of a quantum mechanical treatment may sound obvious since we live in a quantum mechanical world, but it is much more fundamental than one might naively think. Consider a set of geodesics that scan past light cones dense enough at late times, i.e., when conditions  $A$  can be satisfied. Because of the rapid exponential expansion of spacetime, these geodesics must have been much closer at early times—in fact, there are generically a huge number of geodesics emanating from an early space-like hypersurface of the Planck size. On the other hand, theories of quantum gravity suggest that the Planck size region can contain only  $O(1)$  (or smaller) information. How can such a state evolve into many different universes in the future? Is it reasonable to expect that the procedure (ii) above, based on the classical spacetime picture, can select the appropriate ensemble of past light cones, despite the fact that it involves scales shorter than the Planck length at early times?

We find that quantum mechanics plays a crucial role in answering these questions—in fact, the ultimate resolution of the measure problem *requires* quantum mechanical interpretation of the multiverse. Our basic assumption here is that the laws of quantum mechanics—deterministic, unitary evolution of quantum states and the superposition principle—are not violated when physics is described from an observer’s point of view. As we will argue, this implies that the *complete* description of the multiverse can be obtained *purely* from the viewpoint of a single observer traveling the multiverse. All the information on the multiverse is contained in the (stretched) apparent horizon and spacetime therein, as seen from that observer. This situation is similar to describing a black hole from the viewpoint of a distant observer using “complementarity” [7]—indeed the situation for a black hole arises as a special case of our general description. Our construction is strongly motivated by such complementarity view of spacetime, as well as the holographic principle [8].

To exemplify this picture further, let us consider that spacetime was initially in a highly symmetric configuration, e.g. four dimensional de Sitter spacetime, with the fields sitting in a local minimum of the potential. From the semi-classical analysis, we know that even a tiny region of this initial configuration evolves into infinitely many different universes. On the other hand, the holographic principle, or de Sitter entropy, says that such a small region can have only finite degrees of freedom. This implies that in the quantum picture, the origin of various semi-classical universes cannot be attributed to the difference of initial conditions—they must arise as different “outcomes” of a quantum state  $\Psi$ , which is uniquely determined once an initial condition is given. This clearly answers one question raised above: how can an initial state having only  $O(1)$  information evolve into different universes? In fact, the initial state evolves into the *unique* future state, which however is a probabilistic mixture of *different* (semi-classical) universes. The meaning

of the sampling procedure in (ii) also becomes clear—sending a geodesic corresponds to making a “measurement” on  $\Psi$ , and the sampling corresponds to *defining* probabilities through “repeated measurements.” In particular, we need not take too seriously the sub-Planckian distances appearing in the procedure—the semi-classical picture of the multiverse is simply a pictorial way of representing probabilistic processes, e.g. bubble nucleation processes, occurring in the quantum universe.<sup>1</sup>

The remaining task to define a complete, quantum mechanical framework for prediction is to come up with the explicit formalism of implementing procedure (ii), given originally within the semi-classical picture. To do so, we first define  $\Psi$  more carefully. For simplicity, we take it to be a pure state  $|\Psi\rangle$  (although an extension to the mixed state case is straightforward). A crucial point is that we describe the system *from a single observer’s point of view*. This allows us to consider the state  $|\Psi(t)\rangle$ , where the time *parameter*  $t$  is taken as a proper time along the observer, which can be assigned a simple, invariant meaning. (We take the Schrödinger picture throughout.) We consider that  $|\Psi(t)\rangle$  is defined on observer’s past light cones bounded by the (stretched) apparent horizons, as viewed from the observer. This restriction on spacetime regions comes from consistency of quantum descriptions for systems with gravity. If we provide an initial condition on a space-like hypersurface  $\Sigma$ , then the state is given on past light cones and  $\Sigma$  at early times (as in Fig. 2), and later, on past light cones inside and at the horizons (see Figs. 4, 5 and Eq. (21) in Section 4.4).

In general, the state  $|\Psi(t)\rangle$  can be expanded as

$$|\Psi(t)\rangle = \sum_i c_i(t) |\alpha_i\rangle, \quad (2)$$

where  $|\alpha_i\rangle$  span a complete set of orthonormal states in the landscape (i.e. all possible past light cones within the horizons), and the index  $i$  may take continuous values (in which case the sum over  $i$  should be interpreted as an integral). Here, we have taken the basis states  $|\alpha_i\rangle$  to be independent of  $t$ , so that all the  $t$  dependence of  $|\Psi(t)\rangle$  come from those of the coefficients  $c_i(t)$ . Once the initial condition is given, the state  $|\Psi(t)\rangle$  is determined uniquely. (Our framework does not provide the initial condition; rather, it is modular with respect to initial conditions.)

We next introduce an operator  $\mathcal{O}_A$  which projects onto the states consistent with conditions A imposed on the past light cone:

$$\mathcal{O}_A = \sum_i |\alpha_{A,i}\rangle \langle \alpha_{A,i}|, \quad (3)$$

where  $|\alpha_{A,i}\rangle$  represent the complete set of states that satisfy conditions A. The conditions can be imposed either within or on the past light cone, since they can be transformed to each other using (supposedly known) deterministic, quantum evolution. Note that, as long as conditions A

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<sup>1</sup>While completing this paper, I learned that Raphael Bousso also arrived at a similar picture in the context of geometric cutoff measures [9].

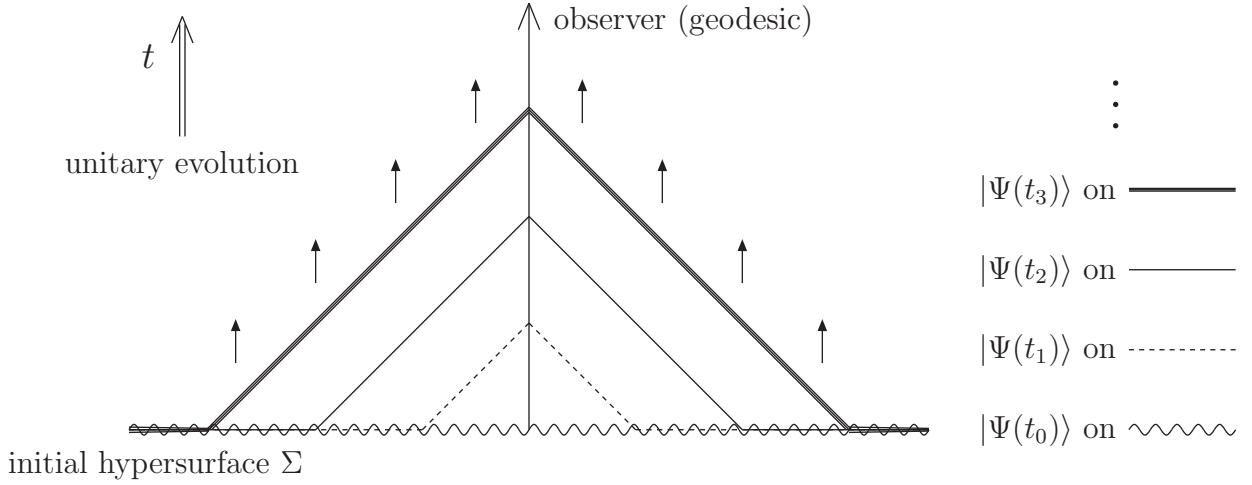


Figure 2: The entire multiverse can be described purely from the viewpoint of a single “observer” in terms of a quantum state  $|\Psi(t)\rangle$ , which is defined on the observer’s past light cones (and on the initial hypersurface  $\Sigma$  at early times; later, the light cones are bounded by the apparent horizons). Once the initial condition is given, the state is uniquely determined according to unitary, quantum mechanical evolution.

are physical (i.e. do not depend on arbitrary parametrization  $t$ ), the operator  $\mathcal{O}_A$  does not depend on  $t$ , so that it can be written in the form of Eq. (3).

The probability that a past light cone satisfying  $A$  also has a property  $B$  is then given by

$$P_{\Sigma}(B|A) = \frac{\int dt \langle \Psi(t) | \mathcal{O}_{A \cap B} | \Psi(t) \rangle}{\int dt \langle \Psi(t) | \mathcal{O}_A | \Psi(t) \rangle} \rightarrow \frac{[\sum_{i,j} |c_i(t_j)|^2]_{A \cap B}}{[\sum_{i,j} |c_i(t_j)|^2]_A}, \quad (4)$$

which is the quantum version of the probability  $P(B|A)$  in Eq. (1). Here, we have put the subscript  $\Sigma$  to remind us that the probability depends on initial conditions. The rightmost expression arises when conditions  $A$  specify an exact “time,” e.g. by requiring a particular value for a parameter that has smooth  $t$  dependence; in such a case, the conditions select definite (discrete) times  $t_j$ . Note that specifying physical “time” is different from specifying time *parameter*  $t$ ; in fact, the same “time” can occur multiple times in the history at  $t = t_j$  ( $j = 1, \dots$ ).

The probability defined in Eq. (4) inherits the following crucial properties from the semi-classical definition:

- **well-defined** — While there are cosmic histories which encounter  $A$ -satisfying past light cones an infinite number of times, they compose only a measure zero set. In particular, histories encountering  $n$  such light cones occur only with  $e^{-cn}$  probabilities with  $c > 0$ , because, starting from any generic state, the probability of encountering  $A$ -satisfying light

cones is exponentially small. This makes the sums (time integrals) in Eq. (4) converge, and so makes  $P_\Sigma(B|A)$  well-defined.

- **“gauge invariant”** — The probability  $P_\Sigma(B|A)$  does not depend on parametrization of time  $t$ , as is clear from the expressions in Eq. (4). The role of time in making predictions can then be played by a physical quantity that has smooth dependence on  $t$  [10], which need not be defined globally in the entire spacetime. In fact, it can be arbitrary information on any physical system, as long as it is not about a static property of the system.

Since quantum evolution of the state is deterministic, the probability for a future event  $C$  to happen,  $P_\Sigma(C|A)$ , is also calculated once the values of  $P_\Sigma(B|A)$  are known for all  $B$ ’s that could affect the likelihood for  $C$  to occur (assuming, of course, that the evolution law is known).

In principle, the definition of Eq. (4) can be used to answer any physical questions associated with prediction and postdiction in the multiverse. However, since  $|\Psi(t)\rangle$  involves degrees of freedom on horizons, as well as those in the bulk of spacetime, its complete evolution can be obtained only with the knowledge of quantum gravity. This difficulty is avoided if we focus on the bulk degrees of freedom, by considering a *bulk density matrix*

$$\rho_{\text{bulk}}(t) = \text{Tr}_{\text{horizon}} |\Psi(t)\rangle \langle \Psi(t)|, \quad (5)$$

where  $\text{Tr}_{\text{horizon}}$  represents the partial trace over the horizon degrees of freedom. This description allows us to make predictions/postdictions without knowing quantum gravity, since the evolution of  $\rho_{\text{bulk}}(t)$  can be determined by semi-classical calculations in low energy quantum field theories. The cost is that (apparent) unitarity violation is induced in processes involving horizons, such as black hole evaporation.

Our framework has a number of implications. Major ones are:

- **The measure problem in eternal inflation is solved.** The principle behind the solution is quantum mechanics, where quantum states are defined in the spacetime region bounded by (stretched) apparent horizons *as viewed from a single “observer” (geodesic)*. No ad hoc cutoff needs to be introduced, and problems/paradoxes plaguing other measures are absent. Transformations between descriptions based on different observers are possible, but they are in general highly nonlocal when the observers are not in causal contact at late times.
- **The multiverse and many worlds in quantum mechanics are the same.** Our framework provides a *unified treatment* of quantum measurement processes and the eternally inflating multiverse, usually associated with vastly different scales—smaller than atomic and much larger than the universe. The probabilities in microscopic quantum processes and in the multiverse are both understood as “branching” in the entire multiverse state  $|\Psi(t)\rangle$  (or

equivalently, as a “misalignment” in Hilbert space between  $|\Psi(t)\rangle$  and the basis of local observables).<sup>2</sup>

- **Global spacetime can be viewed as a derived concept.** By rearranging various terms in the multiverse state  $|\Psi(t)\rangle$ , the global spacetime picture can be “reconstructed.” In particular, this allows us to connect the quantum probabilities with the semi-classical probabilities, defined using geodesics traveling through global spacetime of the multiverse.
- **The multiverse is a transient phenomenon while relaxing into a supersymmetric Minkowski state.** Our framework suggests that the ultimate fate of the multiverse is a supersymmetric Minkowski state, which is free from further decay. The other components in  $|\Psi(t)\rangle$ , e.g. ones hitting big crunch or black hole singularities, simply “disappear.” This leads to the picture that our entire multiverse arises as a transient phenomenon, during the process of some initial state (either determined by quantum gravity or generated as a “fluctuation” in a larger structure) relaxing into a stable, supersymmetric final state.

Other implications are also discussed throughout the paper.

We emphasize that the multiverse state  $|\Psi(t)\rangle$  is literally “everything” in terms of making predictions—even we ourselves appear as *a part of*  $|\Psi(t)\rangle$  at some time(s)  $t_j$ . Once the state is given, any physical predictions can be obtained using Eq. (4), which is nothing but the standard Born rule. There is no need to introduce wavefunction collapse, environmental decoherence, or anything like those—indeed, there is no “external observer” that performs measurements on  $|\Psi(t)\rangle$ , and there is no “environment” with which  $|\Psi(t)\rangle$  interacts. From the viewpoint of a single observer (geodesic), probabilities keep being “diluted” because of continuous branching of the state into different semi-classical possibilities, caused by the fact that the evolution of  $|\Psi(t)\rangle$  is not along an axis in Hilbert space determined by operators local in spacetime. Indeed, the probabilities are constantly being dissipated into (fractal) Minkowski space, which acts as an infinite reservoir of (coarse-grained) entropy—this is the ultimate reason behind the well-definedness of the probabilities. It is quite remarkable that this simple and satisfactory picture comes with a drastic change of our view on spacetime: *the entire world can be described within the spacetime region inside the causal patch of a single geodesic, bounded by its (stretched) apparent horizons*. All the information about the world is contained in a *single* quantum state  $|\Psi(t)\rangle$  defined in this region—we do not even need to consider an ensemble of quantum states. In this sense, we may even say that spacetime exists only to the extent that it can be accessed directly by *the* single “observer” (geodesic).

The organization of this paper is the following. In Section 2, we provide detailed discussions on what prediction and postdiction mean in physical theories, especially in the context of the

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<sup>2</sup>The conjecture that the multiverse and many worlds may be the same thing appeared earlier in Ref. [11]. (I thank Leonard Susskind for bringing this to my attention.) Here we show that they are, in fact, identical, following the same formula for the probabilities, Eqs. (28) and (29). This provides complete unification of the two concepts.

multiverse. We present precise definitions of prediction/postdiction, as well as their practical implementations in realistic contexts. In Section 3, we present our explicit framework for prediction, using the semi-classical picture; but we will also see that the framework cries for a quantum mechanical treatment of the multiverse. Based on these results, in Section 4 we introduce a fully, quantum mechanical framework for prediction. We carefully consider quantum states, and argue that they must be defined in the region within (stretched) apparent horizons, viewed from a single “observer” traveling through the multiverse. We also introduce an approximation scheme in which horizon degrees of freedom are integrated out, so that the probabilities can be calculated without knowing quantum gravity.

The issue of initial conditions is discussed in Section 5, both from the perspective of making predictions in eternally inflating universe and addressing fundamental questions regarding space-time. In Section 6, we present a unified picture of the eternally inflating multiverse and quantum measurement processes—in fact, this unification is an automatic consequence of our treatment of the multiverse. We also discuss a connection of our picture, based on a single “observer,” with the global spacetime picture. In Section 7, we show that the framework does not suffer from problems plaguing other measures proposed so far [5] based on geometric cutoffs; in particular, it avoids a peculiar conclusion that time should “end” [12]. Finally, in Section 8 we summarize what we learned about eternal inflation, and discuss further issues such as the beginning/fate of the multiverse, the possibility of a fully holographic description, and a structure even larger than the entire multiverse (“mega-multiverse”) which is suggested by extrapolating our framework to the extreme.

Appendix A discusses an interpretation of the framework in terms of “fuzzy” time cutoff. Appendix B provides sample calculations of probabilities in toy landscapes, both in the semi-classical and quantum pictures. Appendix C presents an analysis of a gedanken experiment which provides a nontrivial consistency check of our framework.

## 2 What Questions Should We Ask?

In this section, we discuss in detail what are “legitimate” physical questions we would ask in the context of the eternally inflating multiverse. For example, we may want to ask what are the results of future measurements, what is the probability of certain Lagrangian describing our universe, or if the observations we have already made are consistent with the assumption of typicality in the multiverse. We will see that answering these questions is boiled down to coming up with a well-defined prescription for selecting appropriate samples of past light cones (either finite or infinite numbers) that are consistent with our “prior knowledge” about our past light cone. The actual prescription will be given in the next section, where we will see that it naturally calls for a quantum

mechanical treatment of the multiverse.

## 2.1 Predictions in a strict sense

Suppose you want to “predict” what physics you will see at the LHC experiments, assuming you have a perfect knowledge about the underlying theory governing evolution of systems, including the multi-dimensional scalar potential in the string landscape (but, of course, *not* about which vacuum you live). Specifically, you want to know what is the probability for you to find that weak scale supersymmetry, technicolor, just the standard model, or something else, is the Lagrangian describing your local universe, i.e. physics within your Hubble horizon.<sup>3</sup>

In the multiverse context, what this “really” means is the following. First, you specify all you know about your past light cone.<sup>4</sup> This includes the existence of you, the room around you (if you are reading this in a room), the earth, and (the observable part of) the universe. You then ask what is the probability of this “initial situation”—i.e. the particular configuration within your past light cone—to evolve into a certain “final situation”—a past light cone lying in your future in which you will somehow learn that physics at the TeV scale is, e.g., weak scale supersymmetry (either from a friend, paper, ...). In fact, possible past light cones in your future must include ones in which the LHC fails, e.g. due to some accidents, so that no relevant measurement will be made. After including all these possible (mutually exclusive) futures, the probabilities should add up to one.

Of course, you practically do not know everything you need to specify *uniquely* your past light cone. (You even do not know the location of a coffee cup next to you with infinite precision.) This implies that, even at the classical level, the “initial situation” you prepare must be an ensemble of “situations”—i.e. a collection of past light cones—that are consistent with the knowledge you have about your past light cone.<sup>5</sup> Therefore, a specification of the “initial situation” requires some method that assigns relative weights among these “initial past light cones”—namely, you need

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<sup>3</sup>If the LHC has answers to these questions by the time you read this paper, then you should appropriately replace statements below with those regarding some other experiment whose results are not yet available to you.

<sup>4</sup>Here, I consider “you” as a point in spacetime, for simplicity. To be precise, “you” must (at least) be some pattern of electromagnetic activities (neural signals) corresponding to a brain in your “current” moment, which occur inside but around the tip of the light cone. In fact, if you are not a Boltzmann brain, which we assume throughout this section, your entire body (indeed, the entire you from the birth to the present) must exist inside the light cone. For more discussions on these and related issues, see Section 7.1.

<sup>5</sup>In fact, in a classical world, if you have a *complete* knowledge about your past light cone, you would not even ask about future probabilities, since the physics is fully deterministic (assuming the initial conditions at the earliest moment are also known). For example, you would be able to “calculate” if the LHC fails or not, and you would know physics at very high energies through tiny effects encoded in higher dimension operators in the effective theory. This apparently does not sound to be the case in a quantum world, since even if you completely specify your past light cone, outcome of future experiments can only be determined probabilistically. However, for a given initial quantum state, its future states *are* uniquely determined. In fact, the concept of making “predictions” in the sense considered here is relevant only if our knowledge about the current state of the system is incomplete.

to find samples of past light cones that “faithfully” represent incompleteness of your knowledge. In fact, in the global spacetime picture of the multiverse, the number of past light cones that satisfy any input knowledge will be infinite, so some regularization prescription will be needed to define the samples. (This is the measure problem in the eternally inflating multiverse.) On the other hand, once such samples are given, it is *conceptually* a straightforward matter to work out predictions, following the procedure described above.

## 2.2 Predictions in practice

While the definition of predictions described in the previous subsection is “rigorous,” it is not very practical. In practice, when we ask questions, e.g., about physics at the TeV scale, we only want to keep track of global quantities which are (reasonably) uniform in our horizon, such as the CMB temperature, the size of its statistical fluctuation, masses of “elementary” particles, the gauge group, and so on. In this context, how can we make predictions, e.g., about the probability of our local universe being described by a certain Lagrangian? We emphasize that this issue is logically separate from that of defining probabilities; rather, the issue here is to come up with reasonable approximation schemes which we may hope to be tractable.

Suppose you only specify that your past light cone—more precisely the intersection of your past light cone and your own bubble universe—is described by the standard models of particle physics and cosmology at energies below e.g. a TeV, and that the CMB temperature at the tip of the cone is  $T_{\text{CMB}} = 2.725$  K. Here, the latter condition is imposed to specify a physical “time,” which in the previous treatment was done by giving a particular configuration in the past light cone (e.g. the “current” state of your brain; see footnote 4). Then the ensemble you need to prepare as an “initial situation” includes past light cones in which physics at the TeV scale is the minimal standard model, weak scale supersymmetry, technicolor, and so on (as long as these theories arise in consistent vacua in the landscape).

Now, in selecting these light cones, you did *not* impose any condition about yourself. However, as long as the chance that you are born does not depend much on TeV-scale physics, you can expect

$$\frac{\mathcal{N}_{\text{“you” in SM}}}{\mathcal{N}_{\text{SM}}} \sim \frac{\mathcal{N}_{\text{“you” in SUSY}}}{\mathcal{N}_{\text{SUSY}}} \sim \frac{\mathcal{N}_{\text{“you” in technicolor}}}{\mathcal{N}_{\text{technicolor}}} \sim \dots, \quad (6)$$

where  $\mathcal{N}_{\text{“you” in SM}}$  and  $\mathcal{N}_{\text{SM}}$  (and similarly for others) are, respectively, the numbers of past light cones selected *with* and *without* the condition that you are in the light cones (around the tips). In this case, you can use  $\mathcal{N}_{\text{SM}}$ ,  $\mathcal{N}_{\text{SUSY}}$ , and  $\mathcal{N}_{\text{technicolor}}$ , instead of  $\mathcal{N}_{\text{“you” in SM}}$ ,  $\mathcal{N}_{\text{“you” in SUSY}}$ , and  $\mathcal{N}_{\text{“you” in technicolor}}$ , to compute relative probabilities for each TeV-scale physics describing your own universe.<sup>6</sup> Of course, in the context of predicting results at the LHC, “SUSY” should mean

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<sup>6</sup>To compute absolute probabilities, these numbers must be divided by the number of past light cones that are

the existence of superpartners within the reach of the LHC.

Note that when selecting samples for  $\mathcal{N}_{\text{SM}}$ ,  $\mathcal{N}_{\text{SUSY}}$ ,  $\mathcal{N}_{\text{technicolor}}$ , ..., the condition was imposed that physics below a TeV scale is described by the standard models of particle physics and cosmology, with various parameters—such as the electron mass, fine structure constant, dark matter abundance, and baryon-to-photon ratio—taking the observed values (within experimental errors). This is important especially because your own existence may be strongly affected by the values of these parameters. Since some of the parameters, e.g. cosmological ones, will depend highly on physics at high energies, the ratios between  $\mathcal{N}_{\text{SM}}$ ,  $\mathcal{N}_{\text{SUSY}}$ ,  $\mathcal{N}_{\text{technicolor}}$ , ..., are in general *not* the same as the corresponding ratios obtained without conditioning the values of low energy parameters. From the practical point of view, this makes the problem hard(er); but again, once appropriate samples of past light cones are obtained, it is straightforward to make probabilistic predictions for physics at the TeV scale.

## 2.3 Predictions and postdictions

At first sight, it might seem completely straightforward to apply the method described so far to “postdict” some of the physical parameters we already know—we simply need to select samples of past light cones without imposing any condition on those parameters. This, however, needs to be done with care, because our own existence may be affected by the values of these parameters. In the context of an approximation scheme discussed in the previous subsection, appropriate postdictions are obtained after specifying an “anthropic factor”  $n(x_i)$ —the probability of ourselves developing in the universe with parameters taking values  $x_i$ . With this factor, the probability density for us to observe  $x_i$  is given by

$$P(x_i) dx_i \propto \mathcal{N}(x_i) n(x_i) dx_i, \quad (7)$$

where  $\mathcal{N}(x_i)$  is the number of past light cones in which the parameters take values between  $x_i$  and  $x_i + dx_i$ . Note that, when selecting samples for  $\mathcal{N}(x_i)$ , we still need to specify the “time” at which postdictions are made (e.g. through  $T_{\text{CMB, tip}} = 2.725$  K). In fact, the level of the validity of Eq. (7) is determined by how well  $n(x_i)$  and the specification of “time” can mimic ourselves.<sup>7</sup> Postdictions obtained in this way can then be used to test the multiverse hypothesis, by comparing them with the observed data.

A treatment of postdictions analogous to that of predictions in Section 2.1 would require a subtle choice for the condition of selecting light cone samples. Suppose we want to postdict the electron mass,  $m_e$ , and compare it with the experimental value,  $m_{e,\text{exp}} = 510.998910 \pm 0.000013$  keV. We

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selected only with the condition that they are consistent with your current knowledge about the physical laws in your own universe, e.g., constraints from collider and rare decay experiments.

<sup>7</sup>A simple approximation, appropriate in some cases, is obtained by assuming that  $n(x_i) = 1$  for habitable regions while  $n(x_i) = 0$  for regions hostile for life. See e.g. Ref. [13] for more details about this approximation.

would then need to prepare the ensemble of past light cones that have some “consciousnesses” around the tip perceiving similar worlds as we see now *except* that the electron mass may not take the value  $m_{e,\text{exp}}$ . However, since worlds with different electron masses will not be identical, we will not be able to require these consciousnesses to have *identical* thoughts/memories. In this sense, the concept of postdiction will have intrinsic ambiguities associated with the specification of reference observers, and its treatment can only be “approximate,” e.g., in the sense of Eq. (7).

### 3 A Framework for Prediction—Selecting the Ensemble of Past Light Cones

We have argued that, assuming the underlying theory is known, answering any physical questions in the multiverse is boiled down to finding appropriate samples for past light cones that satisfy your input information. This is equivalent to specifying relative weights for various “microscopic” possibilities that are consistent with your input. In this section we provide an explicit prescription for obtaining an appropriate ensemble of past light cone samples. We will focus on the implementation of the idea in a semi-classical world, deferring the complete, quantum mechanical formulation to Section 4. We will, however, see that our prescription naturally demands a quantum mechanical treatment of the multiverse.

#### 3.1 Semi-classical picture

A simple prescription for finding the required ensemble is to define it as the result of “repeated experiments.” In the context of the eternally inflating multiverse, this means that we need to “simulate” many times the entire multiverse as viewed from a single “observer.” Defining this way, the result will depend on the initial setup for the “experiments,” i.e. the initial condition for the evolution of the multiverse. We will discuss this issue in more detail in Section 5, but for now we simply assume that the multiverse starts from some initial state that is highly symmetric. In particular, we assume that we can choose a time variable in such a way that, at sufficiently early times, constant-time space-like hypersurfaces are homogeneous and isotropic.

We now focus on a generic, finite region  $\Sigma$  on an initial space-like hypersurface which contains at least one (measure zero) eternally inflating point. Because of the homogeneity of the hypersurface, the choice of the region can be arbitrary. (If the volume of the initial hypersurface is finite, as in the case where the spacetime is born as a closed universe, then  $\Sigma$  may be chosen to be the entire hypersurface.) We then consider a set of randomly distributed spacetime points  $p_i$  on  $\Sigma$ , and a future geodesic  $g_i$  emanating from each  $p_i$ . Here we choose all the  $g_i$ ’s to be normal to  $\Sigma$ ; with this

choice, the symmetry of the spacetime is recovered in the limit of a large number of  $p_i$ ,  $N_p \rightarrow \infty$ .<sup>8</sup>

Despite the fact that the spacetime is eternally inflating, any of the geodesics  $g_i$  (except for a measure zero subset) experiences only a finite number of cosmic phase transitions and ends up with one of the terminal vacua—i.e. vacua with absolutely zero or negative cosmological constants—or a black hole singularity.<sup>9</sup> This implies that if we follow the “history” of one of these  $g_i$ ’s by keeping track of the evolution of its past light cone, the possibility of finding a light cone that is consistent with any input conditions is extremely small. However, since the input conditions necessarily have some “range,” as discussed in Section 2.1, the probability of finding such a light cone is nonzero,  $P > 0$ .<sup>10</sup> Therefore, if we prepare a large number of  $p_i$ ’s:

$$N_p \gg \frac{1}{P}, \quad (8)$$

then we can find many light cone samples that satisfy the conditions provided. A schematic depiction of this procedure was given in Fig. 1 in Section 1. Note that, if the input involves a specification of an exact physical “time,” e.g. through the precise state of our own brains (Section 2.1) or through  $T_{\text{CMB, tip}}$  (Section 2.2), then the number of light cones selected is countable and finite for each geodesic.<sup>11</sup>

Our proposal is to use the ensemble of past light cones obtained in this way to make predictions/postdictions according to the procedures described in Section 2. Suppose, for example, that we obtain  $\mathcal{N}_A$  past light cone samples satisfying prior conditions  $A$ , and that among these samples  $\mathcal{N}_{A \cap B}$  past light cones also satisfy conditions  $B$ . Then, the conditional probability of  $B$  under  $A$ ,  $P(B|A)$ , is given by

$$\frac{\mathcal{N}_{A \cap B}}{\mathcal{N}_A} \xrightarrow{N_p \rightarrow \infty} P(B|A). \quad (9)$$

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<sup>8</sup>The condition that the symmetry recovers in the large  $N_p$  limit does not uniquely fix the initial condition; in particular, we can have an arbitrary distribution for the magnitude of the  $g_i$  velocities,  $f(|\vec{v}|)$ , as long as their direction  $\vec{v}/|\vec{v}|$  is random. While  $|\vec{v}|$  decays exponentially with time during inflation, this can still affect predictions [14] albeit to a small extent [15]. Here we have chosen  $f(|\vec{v}|) = \delta(|\vec{v}|)$  simply to illustrate our method in a “minimal” setup. The issue of initial conditions in eternally inflating universe will be discussed in Section 5.1. Ultimately, when we apply our framework to the entire multiverse, the initial condition needs to be specified by the “theory of the beginning.” Candidates for such a theory will be discussed in Sections 5.2 and 8.4.

<sup>9</sup>We assume that the landscape has (at least one) terminal vacua, as suggested by string theory. This assumption is also needed to avoid the Boltzmann brain problem; see Section 7.1.

<sup>10</sup>If the landscape is decomposed into several disconnected pieces, some choice for the initial state may lead to  $P = 0$ . This implies that our vacuum does not belong to the same landscape component as the initial state, and we need to (re)choose another initial state to make predictions/postdictions. In order for the entire framework to make sense (i.e. our own existence to be compatible with the multiverse picture), the correct choice for the initial state must lead to  $P > 0$ .

<sup>11</sup>The specification of an exact time is *not* a necessity. Indeed, in a real world we never have an exact knowledge about the current time, so that the “initial situation” light cones have some spread in the time direction, requiring us to count the passing of  $g_i$  through tips of these light cones as a “single event.” In many cases, however, our knowledge about the current time is sufficiently precise for the present purposes, so that the error arising from treating it as an exact time is negligible in the limit where  $N_p$  is large.

Since the distribution of  $p_i$ 's is not correlated with the histories afterwards, the resulting probability does not depend on this distribution in the limit of very large  $N_p$ . Also, since  $\mathcal{N}_A$  and  $\mathcal{N}_{A \cap B}$  are both finite at arbitrarily large  $N_p$ , the probability is well-defined.

Similarly, in order to predict the future in the sense of Section 2.1, we can count the number of past light cones in the sample that evolve into a particular future situation  $C$ :  $\mathcal{N}_{A \rightarrow C}$ . The probability that  $C$  occurs under conditions  $A$  is then given by

$$\frac{\mathcal{N}_{A \rightarrow C}}{\mathcal{N}_A} \xrightarrow{N_p \rightarrow \infty} P(C|A). \quad (10)$$

If we select a set of future situations  $C_i$  which are exhaustive and mutually exclusive, then we have

$$\sum_i P(C_i|A) = 1. \quad (11)$$

In the example of predicting the result of the LHC experiments (as in Sections 2.1 and 2.2), we may choose, e.g.,  $\{C_i\}$  = the LHC will {find just the Higgs boson, find supersymmetry, find technicolor, find any other theory or the result will be inconclusive, fail}.<sup>12</sup>

We note that the validity of our method is not limited to vacua with four spacetime dimensions. Our prior conditions  $A$  can be formulated in any numbers of non-compact and compact spatial dimensions. While it is appropriate in most circumstances to limit our discussions to  $(3+1)$ -dimensional spacetime by integrating out all the effects of extra compact dimensions, there is nothing wrong with applying our procedure to the full, higher dimensional context. Such a treatment may be useful if we want to postdict the number of our large spacetime dimensions to be four (unless the anthropic factor  $n(x_i)$  is trivially zero except for four large dimensions). For the universe we live today, our knowledge about compact dimensions must come from bounds from experimental searches, as well as the structure of the observed physical laws if relations between four-dimensional physics and extra dimensional geometries are known in the landscape. The initial space-like hypersurface  $\Sigma$  may also have arbitrary numbers of non-compact and compact spatial dimensions.

In summary, the probabilities are defined through the sampling of past light cones that satisfy our prior conditions, by following histories of many geodesics  $g_i$  emanating from an initial hypersurface  $\Sigma$ . Note that this procedure corresponds precisely to simulating the entire multiverse many times as viewed from a single “observer.”<sup>13</sup> It is quite satisfactory that this operational way of defining probabilities also gives a well-defined prescription for calculating probabilities we are interested in: Eqs. (9) and (10). Some sample calculations of the probabilities are given for toy landscapes in Appendix B.

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<sup>12</sup>In fact, the two definitions of Eqs. (9) and (10) are not independent, since we can calculate the latter,  $P(C|A)$ , from the former,  $P(B|A)$ , once we know  $P(B|A)$  for all  $B$ 's that could affect  $C$ 's.

<sup>13</sup>The procedure may also be viewed as a sort of “fuzzy” time cutoff in expanding universes; see Appendix A.

### 3.2 Necessity of a quantum mechanical treatment

So far, we have been describing our framework using the language of classical spacetime. We know, however, that our world is quantum mechanical. This raises an important question: does the prescription really make sense in the quantum mechanical world? In particular, can we literally follow the histories of  $g_i$ 's classically, emanating from a small initial region  $\Sigma$ ?

Suppose we want to “scan” our current universe finely enough so that we can find a past light cone that satisfies our input conditions. For example, let us imagine that we need one of the  $g_i$ 's for every  $\sim \mu\text{m}^3$  at the current moment in order to follow the procedure described in Section 2.1. In this case, the average separation between  $g_i$ 's at the time when our observable inflation, i.e. the last  $N$ -fold of inflation, began is roughly

$$e^{-N} \frac{T_0}{T_R} \mu\text{m} \sim 10^{-53} \text{ m} \left( \frac{10^8 \text{ GeV}}{T_R} \right), \quad (12)$$

where  $T_0$  and  $T_R$  are the current and reheating temperatures, respectively. Here, we have ignored any focusing of geodesics, e.g. due to structure formation, for simplicity, and used  $N = 60$  to obtain the final number. We find that the distance obtained is much shorter than the Planck length,  $l_P \simeq 1.62 \times 10^{-35} \text{ m}$ ; in other words, we have a large number of  $g_i$ 's passing a Planck size region at the time when the observable inflation started. The number of  $g_i$ 's per a Planck size region increases even more if we extrapolate the history further back. This implies that we need to consider a huge number of  $p_i$ 's in a Planck size region on  $\Sigma$ , from which  $g_i$ 's emanate.

On the other hand, we expect that at the length scale  $\sim l_P$  gravity becomes strong, so that quantum effects become important even for spacetime.<sup>14</sup> In particular, the holographic principle (or de Sitter entropy) suggests that a Planck size region in de Sitter space can contain at most of  $O(1)$  bits of information. Then, how can we make sense out of histories of many  $p_i$ 's *distributed inside a Planck size region*? This naturally suggests the following interpretation of our procedure. Starting from a small (even a Planck size) region in the early universe, we follow its evolution quantum mechanically. Because of the quantum nature, this probabilistically leads to many cosmic histories; and it is these possible histories that correspond to sets of past light cones associated with various  $g_i$ 's in the semi-classical picture.

In the next section, we will see that this interpretation is, in fact, *forced* by the laws of quantum mechanics. Interestingly, quantum mechanics cannot be viewed as “small corrections” to the classical picture—*quantum mechanics is essential to (correctly) interpret the multiverse*.

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<sup>14</sup>If the theory has a large number of species  $n$ , the scale where quantum gravity becomes important is actually  $\sim l_P \sqrt{n}$  (or the string length  $l_s$ ) [16]. This, however, does not affect any of our discussions, including the one here. In the rest of the paper, we will set  $n \approx O(1)$  (or  $l_s \sim l_P$ ) for simplicity, but it is straightforward to recover the dependence on  $n$ , if needed.

## 4 Multiverse as a Quantum Mechanical Universe

In this section, we show that the quantum mechanical interpretation of the multiverse is unavoidable if we assume that the laws of quantum mechanics are not violated when physics is described from an observer’s viewpoint. In particular, we argue that the *entire* multiverse can be described from the point of view of a “single observer.” We develop an explicit quantum mechanical formalism for making predictions/postdictions, which corresponds to the framework described in Section 3 in the semi-classical picture. We first consider the case in which the multiverse is in a pure quantum state, and then generalize it to the mixed state case. We also discuss how the quantum-to-classical transition is incorporated in our framework.

### 4.1 Quantum mechanics and the “multiverse”

Recall the laws of quantum mechanics—deterministic, unitary evolution of the states and the superposition principle. (We adopt the Schrödinger picture throughout.) These laws say: (i) given a pure quantum state  $\Psi(t_0)$ , its history is uniquely determined by solving quantum evolution equation,  $\Psi(t) = U(t, t_0)\Psi(t_0)$  where  $U(t, t_0)$  is a unitary operator; and (ii) if  $\Psi_1(t)$  and  $\Psi_2(t)$  are both solutions of the evolution equation, then  $c_1\Psi_1(t) + c_2\Psi_2(t)$  is also a solution for arbitrary, complex numbers  $c_1$  and  $c_2$ .

While these properties are manifest in the usual formulation of non-relativistic quantum mechanics, they are sometimes obscured in quantum field theory—can’t an initial  $e^+e^-$  “state” evolve into  $e^+e^-$ ,  $\mu^+\mu^-$  or some other “states”? Such an evolution, however, does not contradict the fact that quantum evolution is deterministic. What is happening is simply that the state that is initially  $e^+e^-$ ,  $|e^+e^- \rangle_{\text{in}}$ , evolves into a *unique* final state that has nontrivial overlaps with the states approaching  $e^+e^-$ ,  $\mu^+\mu^-$ ,  $\dots$  in the future,  $|e^+e^- \rangle_{\text{out}}$ ,  $|\mu^+\mu^- \rangle_{\text{out}}$ ,  $\dots$ . Expanding in terms of the states in free field theories (which is possible at  $t \rightarrow \pm\infty$ ), we can write

$$\Psi(t = -\infty) = |e^+e^- \rangle \rightarrow \Psi(t = +\infty) = c_e |e^+e^- \rangle + c_\mu |\mu^+\mu^- \rangle + \dots, \quad (13)$$

where  $c_e = {}_{\text{out}}\langle e^+e^- | e^+e^- \rangle_{\text{in}}$ ,  $c_\mu = {}_{\text{out}}\langle \mu^+\mu^- | e^+e^- \rangle_{\text{in}}$ ,  $\dots$  are uniquely determined.<sup>15</sup> In fact, the evolution of states in quantum field theory is unitary, and satisfies the superposition principle.

These properties of quantum mechanics essentially force us to take the quantum mechanical view of the multiverse, outlined at the end of Section 3.2. Consider a small initial region  $\Sigma$  at an early universe,  $t = t_0$ , which contains an eternally inflating point. If we follow the evolution of this initial (presumably highly symmetric) state quantum mechanically, we should find that future states are uniquely determined. On the other hand, the semi-classical picture of eternal inflation says that various observers emigrating from  $\Sigma$  will see different semi-classical histories, or

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<sup>15</sup>Here we have suppressed the momenta and spins of the particles, but including them is straightforward.

universes. Note that, assuming the energy density on  $\Sigma$  is (much) smaller than the Planck density, we do not have any reason to doubt this semi-classical picture. The only way to reconcile these two facts is to consider that various semi-classical histories correspond to possible outcomes obtained from a unique quantum state  $\Psi(t)$ . Schematically,

$$\Psi(t = t_0) = |\Sigma\rangle \quad \rightarrow \quad \Psi(t) = \sum_i c_i |\text{cosmic history } i \text{ at time } t\rangle. \quad (14)$$

Below, we present more precise arguments on this point and give an explicit formalism which makes well-defined predictions/postdictions possible in the multiverse.

## 4.2 Lessons from black holes

The two expressions in Eqs. (13) and (14) look very similar, but there is one significant difference. While the usual “in-out” formalism of quantum field theory, exemplified in Eq. (13), deals only with the states at infinite past or future (which is indeed quite sufficient for the purpose of describing experiments involving scattering or decay), the cosmological setup of Eq. (14) requires states defined throughout the whole history. As a relativistic version of quantum mechanics, quantum field theory must be able to describe such a state,  $\Psi(t)$ . However, a naive implementation of this under the existence of gravity could cause a problem—the laws of quantum mechanics may be violated.

To see what might happen, let us consider a traveler falling into a black hole, carrying some information. For this traveler, the information is always with him/her until he/she hits the singularity; in particular, it will be inside the black hole at late times. On the other hand, for a distant observer, the information appears to be absorbed into the (stretched) horizon, and then sent back in Hawking radiation. This apparently indicates that the same information exists in two different locations, contradicting the no-cloning theorem of quantum mechanics, which says that a faithful duplication of quantum information is not possible (see Fig. 3).<sup>16</sup> This paradox, however, can be resolved by the so-called black hole complementarity [7]. Since one cannot be *both* a distant observer *and* the falling traveler at the same time, it is perfectly consistent that the two descriptions disagree about something that cannot *in principle* be compared; in particular, they may not agree about where the information exists after the traveler crossed the horizon. When physics is described consistently by a distant observer, or by the falling traveler, the violation of quantum mechanical principles does not occur.

One can consider this phenomenon to arise as a result of noncommutativity of local measurements performed by the two experimenters: the traveler  $T$  and distant observer  $D$ . Suppose that  $T$  and  $D$  have their own complete sets of operators  $\mathcal{T}_i$  and  $\mathcal{D}_i$ , and that we take a Hilbert space

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<sup>16</sup>The no-cloning theorem can be proved using the laws of quantum mechanics stated in Section 4.1.

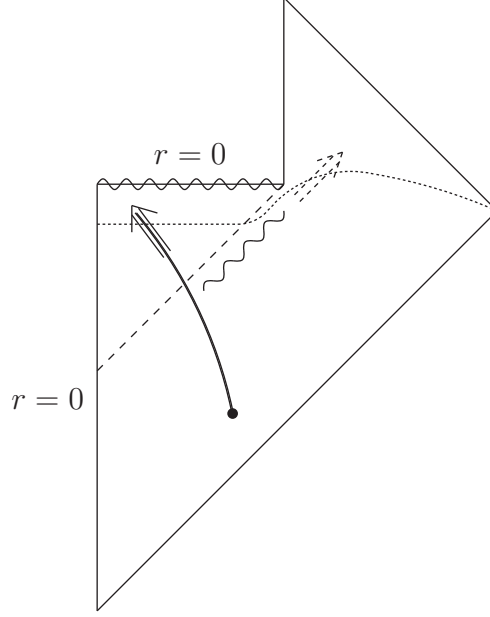


Figure 3: A Penrose diagram representing a traveler who falls into an evaporating black hole (solid curve) carrying some information. For the traveler, information appears to be always with him/her (solid arrow), while from a distant observer, the information appears to be sent back from the black hole in Hawking radiation (dashed arrow). An example of “wrong” constant time hypersurfaces is depicted with the dotted line.

basis in which the states that appear local to  $T$  can be written schematically in the form

$$\psi_1 = \bigotimes \begin{pmatrix} \psi \\ 0_v \\ \vdots \end{pmatrix}, \quad \psi_2 = \bigotimes \begin{pmatrix} 0_v \\ \psi \\ \vdots \end{pmatrix}, \quad \dots, \quad (15)$$

where the symbol  $\otimes$  means that  $\psi_i$  is given by the direct product of all the elements in the vector. Here, each element corresponds to a different spatial point viewed from  $T$ , and  $\psi$  and  $0_v$  represent excited and unexcited states at each point. The local operators for  $T$  then take the (block-)diagonal form:

$$\mathcal{T}_1 = \begin{pmatrix} \hat{\phi} & 0 & \dots \\ 0 & \mathbf{1} & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}, \quad \mathcal{T}_2 = \begin{pmatrix} \mathbf{1} & 0 & \dots \\ 0 & \hat{\phi} & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}, \quad \dots, \quad (16)$$

(ignoring any exponentially damping tails). In the same Hilbert space basis, however, operators

local for  $D$  are *not* necessarily diagonal, e.g.

$$\mathcal{D}_1 = \frac{1}{2} \begin{pmatrix} \mathbf{1} + \hat{\phi} & -\mathbf{1} + \hat{\phi} & \cdots \\ -\mathbf{1} + \hat{\phi} & \mathbf{1} + \hat{\phi} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}, \quad \mathcal{D}_2 = \frac{1}{2} \begin{pmatrix} \mathbf{1} + \hat{\phi} & \mathbf{1} - \hat{\phi} & \cdots \\ \mathbf{1} - \hat{\phi} & \mathbf{1} + \hat{\phi} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}, \quad \dots \quad (17)$$

The states in Eq. (15) may then look highly delocalized for the distant observer  $D$ . (This happens at a time after the traveler passed the horizon). In fact, we expect that (local) information inside the black hole are stored in long-range correlations in Hawking radiation (or in horizon degrees of freedom) from an outside observer point of view [7].

Now, if we want to discuss physics as viewed from distant observer  $D$ , it is absurd to take the basis in Eq. (17)—we should go to the basis where local operators for  $D$  are diagonalized. In the new basis, Eq. (17) becomes

$$\mathcal{D}_1 = \begin{pmatrix} \hat{\phi} & 0 & \cdots \\ 0 & \mathbf{1} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}, \quad \mathcal{D}_2 = \begin{pmatrix} \mathbf{1} & 0 & \cdots \\ 0 & \hat{\phi} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}, \quad \dots, \quad (18)$$

and the states in Eq. (15) look like

$$\psi_1 = \begin{pmatrix} \otimes \left( \begin{matrix} (0_v + \psi)/\sqrt{2} \\ (0_v - \psi)/\sqrt{2} \\ \vdots \end{matrix} \right) \\ \psi_2 = \begin{pmatrix} \otimes \left( \begin{matrix} (0_v + \psi)/\sqrt{2} \\ (-0_v + \psi)/\sqrt{2} \\ \vdots \end{matrix} \right) \\ \dots \end{pmatrix} \quad (19)$$

Two equations (15) and (19) clearly show that when we describe physics from an experimenter's point of view, the two experimenters  $T$  and  $D$  describe the same system very differently—one localized and the other delocalized. (Of course, in the Hilbert space, the  $\psi_i$ 's in the two equations *are* the same states: they simply correspond to different coordinate representations.) Below, when we talk about an observer, we always assume that we take the basis such as Eqs. (16) and (18), i.e. the basis where local operators are “diagonalized,” which is the usual basis for local quantum fields.<sup>17</sup> This means, in particular, that when we change the viewpoint, we must in general perform the associated basis change in the Hilbert space.

The black hole example discussed here also tells us that the choice of constant time slices—a set of hypersurfaces on which quantum states are defined—is extremely important in quantum theories with gravity. Suppose we want to describe the system from a distant observer's point of view, using the local operator basis as described above. How should we define the states? From Fig. 3, it is evident that if we choose a “wrong” time slice (often called a “nice” slice), we would

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<sup>17</sup>Later we will quantize the system on null hypersurfaces, in which case Schrödinger picture operators at different points do not all commute. This subtlety, however, does not affect the basic picture; see Section 4.5.

have a state in which quantum information is duplicated, contradicting the no-cloning theorem. In fact, choosing this kind of slices, nonlocal effects from quantum gravity do not decouple at low energies, and we cannot simultaneously “diagonalize” operators localized in different spatial points [17]. Therefore, if we want to maintain a local description of physics, such choices of time slices should be avoided. The usual in-out formalism of quantum field theory bypasses this issue by considering only states at  $t = \pm\infty$  in asymptotically Minkowski (or anti de Sitter) spacetime, but it is in general important in the context of the cosmology of the multiverse.

### 4.3 Quantum observer principle—relativistic quantum mechanics for cosmology

We now present our quantum mechanical framework for describing the multiverse. Our central postulate, which we may call the *quantum observer principle*, is the following:

*Physics obeys the laws of quantum mechanics when described from the viewpoint of an “observer” (geodesic) traveling the multiverse, although this need not be the case if described in other ways, e.g., using the global spacetime picture with synchronous time slicing. The description involves only spacetime regions inside the (stretched) apparent horizons, as well as the degrees of freedom associated with these horizons.*

As we discussed in the previous subsection, we choose a Hilbert space basis which “diagonalizes” local operators as viewed from the observer; in particular, we take *the same operator basis* for all observers. (We can always do this because, being state independent, complete operator sets for observers are all isomorphic.) To realize the framework explicitly, we still need to define quantum states carefully, which will be discussed in the next subsection. However, without such a realization, the single principle stated above already leads, together with the usual statistical interpretation of horizon entropies, to many nontrivial, general consequences for the description of the multiverse.

Let us now discuss implications of each element of the laws of quantum mechanics in turn:

- **Deterministic evolution** — Consider a small, eternally inflating space-like region  $\Sigma$  at some early time. According to the semi-classical picture, observers in this region can see infinitely many different universes in the future (i.e. the entire multiverse), even if they are “equivalent” (i.e. related by an element of the kinematic subgroup of the de Sitter group). On the other hand, the quantum observer principle says that quantum evolution of states is deterministic; and the holographic principle (or de Sitter entropy) says that the amount of information  $\Sigma$  can carry, i.e. the number of different initial conditions one can prepare on  $\Sigma$ , is finite. This implies that in the quantum picture, the origin of various semi-classical universes cannot be

attributed to the difference of initial conditions on  $\Sigma$ .<sup>18</sup> The only possible interpretation, then, is that these different universes arise as possible outcomes obtained from a quantum state  $\Psi(t)$ , which is uniquely determined once an initial condition is given at some early moment.<sup>19</sup>

- **Unitarity** — The quantum state of the multiverse  $\Psi(t)$  is meaningful if (and, presumably, only if) it is described from the point of view of an observer. In particular, this implies that the state  $\Psi(t)$  is *not* defined outside horizons (of de Sitter, black hole, or any other kind). On the other hand, in the semi-classical picture, there are certainly processes which carry information to outside these horizons. How can unitary evolution of  $\Psi(t)$  be ensured then? The answer is that, as in the black hole case, this information is stored in the stretched horizons when seen by the observer. (Indeed, there is evidence that any cosmological horizons virtually act as black hole horizons; see e.g. [21, 22, 23].) Namely, assuming a pure initial state, subsequent evolution of the multiverse is described by a pure quantum state  $\Psi(t)$  if we include the microscopic description of the stretched horizons. For such a state, the usual, thermal description of a horizon emerges only after considering a suitable statistical ensemble in the multiverse, by “coarse-graining” the horizon degrees of freedom.<sup>20</sup> A nontrivial consistency check of the statement made here is provided in Appendix C.
- **Superposition principle** — Suppose we want to compute the probability of an initial situation  $A$  at time  $t$  to develop into a future situation  $C$  at later time  $t + \Delta t$ . We would then evolve the state representing  $A$  by  $\Delta t$ , and take the overlap with the state(s) corresponding to  $C$ :  $P(C|A) = |\langle \psi_C | e^{-i\hat{H}\Delta t} | \psi_A \rangle|^2$ , where  $\hat{H}$  is the Hamiltonian describing the dynamics of the system. On the other hand, in the description of the entire multiverse, the situation  $A$  occurs multiple times at  $t = t_i$  ( $i = 1, 2, \dots$ ) as one of the possible universes:  $\Psi(t_i) = c_i |A\rangle + \dots$ , where  $c_i \lll 1$  in general. Because of the superposition principle, however, the evolution of the  $c_i |A\rangle$  term in  $\Psi(t_i)$  is exactly the same as the state  $|\psi_A\rangle$  (if we use the complete multiverse evolution operator for  $\hat{H}$ , which is most accurate though often overkill). This implies that we may compute the probability  $P(C|A)$  using the entire multiverse state  $\Psi(t)$ , instead of  $|\psi_A\rangle$ ; in particular, we need not consider that  $\Psi(t)$  collapses to  $|\psi_A\rangle$  when we compute  $P(C|A)$ .

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<sup>18</sup>Mathematically, the de Sitter group cannot be exact at the quantum level, because of the discreteness of energy levels [18]. However, since typical level spacing is of order the inverse Poincaré recurrence time, it is plausible that the effect is physically irrelevant, and semi-classically equivalent observers see equivalent quantum systems. (See also [19].) Indeed, string theory suggests that lifetimes of de Sitter vacua are (much) shorter than the recurrence times [20], so that no energy measurement can physically resolve the discreteness of the levels.

<sup>19</sup>Note that, taking the same operator basis for all observers, the two descriptions in Fig. 3, i.e. by the traveler  $T$  and distant observer  $D$ , correspond to having *different* (though equivalent) states  $\Psi_T$  and  $\Psi_D$ , subjected to different initial conditions: one falling and the other not falling into the black hole. On the other hand, if the two experimenters  $T$  and  $D$  arise in the future of an eternally inflating region  $\Sigma$ , then the two descriptions become parts of a *single* multiverse state,  $\Psi \sim c_T |T\text{'s view}\rangle + c_D |D\text{'s view}\rangle + \dots$ , in the complete description of the multiverse.

<sup>20</sup>For a generic state  $\Psi(t)$ , the entanglement entropy between the bulk and stretched-horizon degrees of freedom is smaller than the full horizon entropy, which takes into account all *possible* microstates associated with the horizon.

An important consequence of the framework described here is that the quantum state  $\Psi(t)$ , defined from a *single* observer’s viewpoint, describes the *entire* multiverse. This is indicated by the fact that eternal inflation populates the entire landscape in the semi-classical picture, even starting from a measure zero point on an initial hypersurface.<sup>21</sup> Namely, *the state  $\Psi(t)$  (which may in general be a pure or mixed state) provides a complete description of the multiverse.*

#### 4.4 The entire multiverse from the viewpoint of a single observer

How can the multiverse state  $\Psi(t)$  be defined explicitly? Let us suppose, for simplicity, that the multiverse is described by a pure state  $|\Psi(t)\rangle$ . (An extension to the mixed state case is straightforward and will be discussed later.) From an observer’s point of view, the subsequent evolution of this state is uniquely determined according to the laws of quantum mechanics. A question is: what constant time slices should we choose to describe the system “as viewed from an observer”? In order to maintain locality of the description, such slices should not extend to the region that cannot be accessed by the observer.

Let us consider how we can define time slices in an operationally well-defined manner. In the multiverse, one obviously cannot carry a physical clock through (some of) the cosmic phase transitions occurring in the landscape—even low energy degrees of freedom may change across bubble walls associated with the transitions. This implies that we cannot use any physical quantities, e.g. average CMB temperature, to define equal time hypersurfaces *throughout the entire multiverse*. Of course, we can adopt any time parametrization *along the geodesic (observer)*—a natural choice is the proper time, which can be assigned a simple, invariant meaning. But how can we extend it to the rest of spacetime without referring to any physical quantities?

We use the causal structure to determine the equal time hypersurfaces, i.e. the hypersurfaces on which quantum states are defined. While this is not an absolute necessity (see discussion in Section 4.5), it provides the conceptually simplest formulation of our framework. Specifically, consider a spacetime point  $p(t)$  along the geodesic (observer) corresponding to a proper time  $t$ . We then define a quantum state  $|\Psi(t)\rangle$  on the past light cone whose tip is at  $p(t)$ . We assume that  $|\Psi(t)\rangle$  is defined only within the stretched horizon, which is located roughly  $l_P$  proper distance in front of the real, mathematical horizon in the static coordinates. The rationale behind this is the following. In the “coarse-grained picture,” which considers an ensemble of spacetime regions having the same macroscopic properties, the local temperature on the stretched horizon is of  $O(1/l_P)$ , due to the blueshift effect. Since this thermal entropy already saturates the horizon entropy, the number of physical degrees of freedom behind the stretched horizon should be at most of  $O(\mathcal{A}_{\text{horizon}}/l_P^2)$ , where  $\mathcal{A}_{\text{horizon}}$  is the horizon area (see, e.g., [24]). This implies that we can describe any physics

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<sup>21</sup>Here we have assumed that the multiverse is irreducible, i.e. any two points in the landscape are connected by some physical processes. The case with a reducible landscape will be mentioned in Section 5 (in footnote 26).

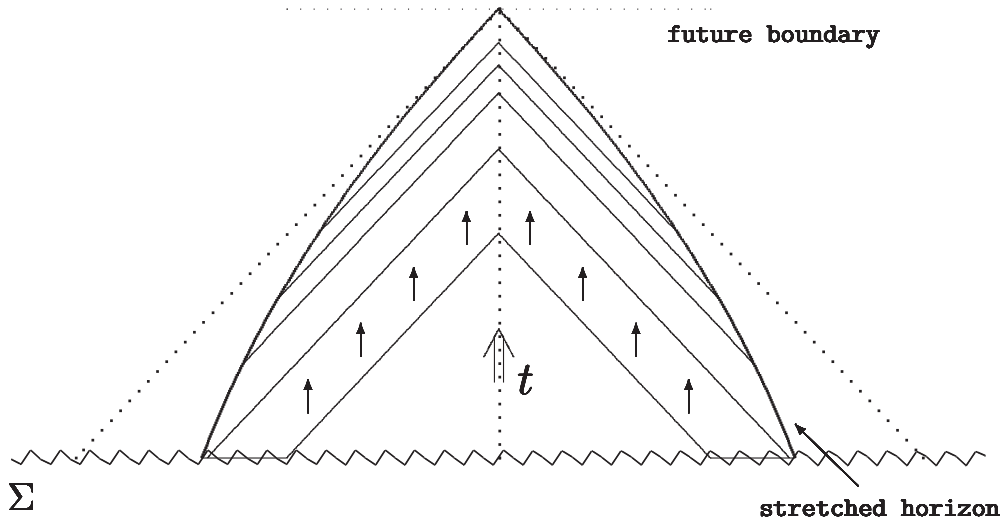


Figure 4: The state  $|\Psi(t)\rangle$  is defined on the past light cones (thin solid lines) bounded either by the initial hypersurface  $\Sigma$  (wavy line) or the stretched horizon (thick solid). The tips of these light cones are on the geodesic corresponding to the “observer,” and the time parameter  $t$  is chosen such that it agrees with the proper time associated with the observer. If the initial condition is given on a past light cone and the stretched horizon bounding it, then the introduction of the space-like hypersurface  $\Sigma$  is not necessary.

in that region using degrees of freedom “on” the stretched horizon, which we include in  $|\Psi(t)\rangle$ .

Suppose that an initial condition for the multiverse is given on a space-like hypersurface  $\Sigma$  (which need not be the case, as we will see shortly). In this case, the multiverse state  $|\Psi(t)\rangle$  is defined at early times on the past light cone within  $C(t)$  and on  $\Sigma$  outside  $C(t)$ , where  $C(t)$  is the intersection between the light cone and  $\Sigma$  (see Fig. 2 in Section 1). This situation, however, does not last long. After a brief initial period, specifically for  $\Delta t \gtrsim -H^{-1} \ln(l_P H)$  after the initial moment, where  $H$  is the Hubble parameter, the state  $|\Psi(t)\rangle$  is given entirely on the past light cones bounded by the stretched horizon, as depicted in Fig. 4.

One may wonder what happens if the observer enters into a Minkowski or anti de Sitter bubble at a late stage in the multiverse evolution. In this case, the (stretched) event horizon disappears, which would be bounding the region where  $|\Psi(t)\rangle$  is defined. Does this mean that we need to know the entire (future) cosmic history to determine the region where  $|\Psi(t)\rangle$  is defined? Or should we suddenly change the defining region of  $|\Psi(t)\rangle$  to the entire past light cone *all the way* back to  $\Sigma$ , when the observer undergoes a transition from a de Sitter to a Minkowski/anti de Sitter phase? Both of these would be highly unreasonable. Considerations along these lines lead us to the following picture: the defining region for  $|\Psi(t)\rangle$  is determined not by the event horizon but by

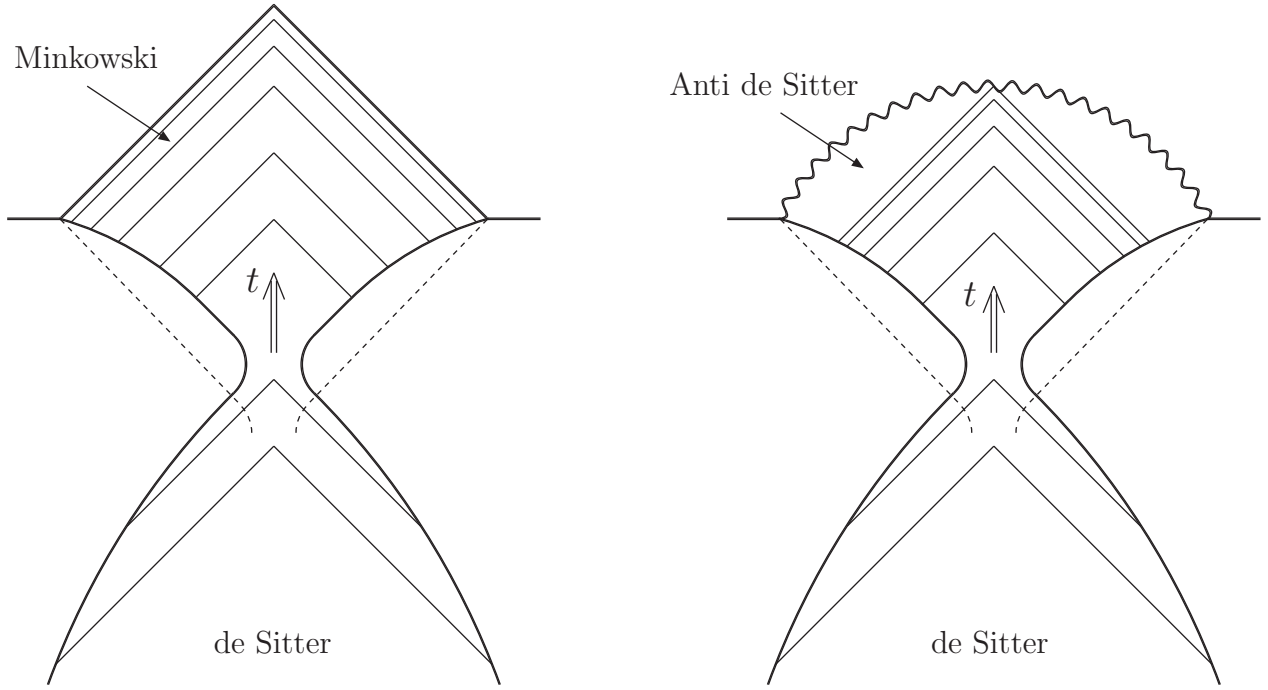


Figure 5: A quantum state for the multiverse,  $|\Psi(t)\rangle$ , is defined on past light cones ( $45^\circ$  lines) bounded by apparent horizons (thick solid lines). The left (right) diagram represents a nucleation of a Minkowski (anti de Sitter) bubble in a meta-stable de Sitter vacuum. The bubble walls are depicted by dashed lines.

the *apparent horizon*, a surface on which the (local) expansion of the cross sectional area of the past light cone turns from positive to negative.<sup>22</sup> With this definition, the state  $|\Psi(t)\rangle$  lives on a smaller portion of the light cone, as sketched in Fig. 5 for nucleations of Minkowski and anti de Sitter bubbles. Note that since the apparent horizon agrees with the Hubble horizon in the limit of small curvature, the picture of Fig. 4 is essentially unchanged. In particular, the horizon should still be stretched.

In fact, there are several reasons to choose the apparent horizon as the boundary of the defining region for the multiverse state:

- The concept of apparent horizon does not depend on low energy physics, such as the gauge group or matter content, so that the definition can be extended to the entire multiverse.
- While the apparent horizon does not exist for all spacetimes, it does exist in *any* Friedmann-Robertson-Walker (FRW) universe that starts from a big-bang or bubble nucleation in a

<sup>22</sup>For a use of apparent horizons in the context of a geometric cutoff, see [25].

higher energy meta-stable vacuum, i.e., spacetime relevant for cosmology in the multiverse.

- The location of an apparent horizon is determined “locally” within the bubble universe. In particular, unlike a particle or event horizon, it does *not* depend on what happens in the infinite past or future. This is very comfortable. In particular, it allows us to describe physics without knowing the future.
- The apparent horizon plays a role of the preferred screen in the holographic principle [26], i.e., its area bounds the amount of entropy on a light sheet associated with it. Here, a light sheet is defined as a congruence of light rays orthogonal to the screen whose cross sectional areas are non-increasing in the direction away from it.

The last point is especially interesting, since our constant time slice is given by a past light cone, which is precisely one of the light sheets associated with the horizon. This allows us to state the holographic principle in a very simple form: *at a given time  $t$ , the maximal number of degrees of freedom in the bulk is bounded by the area of the boundary in Planck units.* Namely,

$$\ln \{ \max \dim (|\Psi_{\text{bulk}}(t)\rangle) \} \leq \frac{1}{4l_P^2} \mathcal{A}_{\text{horizon}}(t), \quad (20)$$

where  $|\Psi_{\text{bulk}}(t)\rangle$  is the bulk quantum state, and  $\mathcal{A}_{\text{horizon}}(t)$  the area of the horizon bounding the “space” (i.e. the light cone). Another important point is that the mandatory existence of the apparent horizon (the second item above) allows us to specify the initial condition on a stretched apparent horizon and the past light cone bounded by it. This has an advantage that the introduction of a space-like hypersurface  $\Sigma$  is not necessary, which considerably simplifies the formalism. In fact, by specifying initial conditions this way, the state  $|\Psi(t)\rangle$  is defined *consistently* on the past light cone bounded by the stretched apparent horizon.

We now have the multiverse state  $|\Psi(t)\rangle$  which is consistent with the principles of quantum mechanics. In general,  $|\Psi(t)\rangle$  consists of a superposition of different semi-classical spacetime configurations, and the above analysis applies to each of these components. The Hilbert space for the multiverse state is, therefore, given by

$$\mathcal{H} = \oplus_{\mathcal{M}} (\mathcal{H}_{\mathcal{M},\text{bulk}} \otimes \mathcal{H}_{\mathcal{M},\text{horizon}}), \quad (21)$$

where  $\mathcal{H}_{\mathcal{M},\text{bulk}}$  and  $\mathcal{H}_{\mathcal{M},\text{horizon}}$  represent Hilbert spaces for the degrees of freedom on the past light cones *inside* and *on* the stretched apparent horizon, respectively, for a fixed semi-classical configuration  $\mathcal{M}$ . The representation of  $\mathcal{H}$  in Eq. (21) is analogous to the Fock space in usual quantum field theories.

How does the multiverse state  $|\Psi(t)\rangle$  evolve? The quantum observer principle states that the evolution is deterministic and unitary:

$$|\Psi(t_1)\rangle = U(t_1, t_2) |\Psi(t_2)\rangle, \quad (22)$$

where  $U(t_1, t_2)$  is a unitary operator acting on the multiverse state  $|\Psi(t)\rangle$ . How can we determine the form of  $U(t_1, t_2)$ ? Since the definition of the state involves horizon degrees of freedom, an explicit expression of  $U(t_1, t_2)$  is obtained only with knowledge of quantum gravity.<sup>23</sup> Therefore, while the description in terms of  $|\Psi(t)\rangle$  is essential for the understanding of the conceptual issues, it is not very “practical” in making predictions for low energy physics (with current theoretical technology). This difficulty, however, can be circumvented if we adopt the following “approximation.” Suppose we are maximally ignorant about the state of the horizon degrees of freedom, so the multiverse is described by a *bulk density matrix*

$$\rho_{\text{bulk}}(t) = \text{Tr}_{\text{horizon}} |\Psi(t)\rangle \langle \Psi(t)|, \quad (23)$$

where  $\text{Tr}_{\text{horizon}}$  means the partial trace over the horizon degrees of freedom. This corresponds to the usual statistical description of the horizons, so that the evolution of  $\rho_{\text{bulk}}(t)$  can be determined by semi-classical calculations in low energy quantum field theories (for example, through thermal treatments for future horizons [27, 21] and stochastic approaches for past horizons [28]). Note that the unitarity of the evolution is *not* preserved in this description, so that information *appears* to be lost in some processes, as was found in the classic black hole analysis by Hawking [29]. The unitarity, however, is recovered once we include the horizon degrees of freedom, as in Eq. (22).

We finally mention the possibility that the multiverse itself is in a mixed state, rather than a pure state (which occurs, e.g., if the theory of initial conditions requires it or if our knowledge of the system is incomplete). In this case, the multiverse is described by a density matrix

$$\rho(t) = \sum_i \lambda_i |\Psi_i(t)\rangle \langle \Psi_i(t)|, \quad (24)$$

whose evolution is given by

$$\rho(t_1) = U(t_1, t_2) \rho(t_2) U(t_2, t_1), \quad (25)$$

where we take  $\sum_i \lambda_i = 1$  following the convention. Again, quantum gravity is needed to obtain the complete evolution. The description corresponding to Eq. (23) can be obtained by considering a reduced density matrix

$$\rho_{\text{bulk}}(t) = \text{Tr}_{\text{horizon}} \rho(t), \quad (26)$$

whose evolution can be determined using semi-classical calculations.

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<sup>23</sup>In particular, quantum gravity must provide a picture of how unitarity is ensured despite the fact that some of the semi-classical histories represented in  $|\Psi(t)\rangle$  end up with big crunch or black hole singularities; a conjecture on this will be given in Section 8.1. Also, since the apparent horizon can have space-like world volumes in some regions, its “time evolution” should be into space-like directions in those regions. This is not a problem. Since the holographic principle allows the apparent horizon to encode all the information on the portion of the observer’s past light cone that lies *in the past of* the horizon, such a “space-like evolution” can be equivalent to the standard time evolution of the system *outside* the horizon.

## 4.5 Probabilities in the quantum universe

Having defined states, we now consider operators. As discussed in Section 4.2, we take the Hilbert space basis in which local operators are “diagonalized.” Strictly speaking, with quantization on past light cones, operators at different points do not all commute, as in the case of usual light front quantization [30]. For example, in an ordinary Minkowski space, operators in the same angular direction do not necessarily commute due to possible causal connections. This subtlety, however, is not essential. In fact, we can avoid it if we adopt quantization on space-like hypersurfaces, instead of null hypersurfaces. The only crucial thing for our framework is to restrict the spacetime region to inside the causal patch bounded by apparent horizons—with this restriction, the problem discussed in Section 4.2 does not arise, and our previous formulae all persist. Here, however, we keep using past light cones as our “equal time” hypersurfaces. This has an advantage that observational conditions are easier to impose, since our “direct” knowledge (without using the evolution equation) is intrinsically limited to the spacetime region inside our past light cone.

The probabilities are defined through operators projecting Hilbert space onto subspaces which satisfy specified (observational) conditions. Let  $|\Psi_{A,i}\rangle$  be a set of orthonormal states that satisfy condition  $A$ . Then the corresponding projection operator is given by

$$\mathcal{O}_A = \sum_i |\Psi_{A,i}\rangle \langle \Psi_{A,i}|. \quad (27)$$

The probability that a past light cone satisfying  $A$  also has a property  $B$  is then given by

$$P(B|A) = \frac{\int dt \langle \Psi(t) | \mathcal{O}_{A \cap B} | \Psi(t) \rangle}{\int dt \langle \Psi(t) | \mathcal{O}_A | \Psi(t) \rangle}, \quad (28)$$

where we have assumed that the multiverse is in a pure state  $|\Psi(t)\rangle$ . This probability corresponds to the semi-classical probability given in Eq. (9). We can similarly define the probability of a past light cone  $A$  to evolve into a particular future situation  $C$ :

$$P(C|A) = \frac{\int dt \langle \Psi(t) | \mathcal{O}_{A \rightarrow C} | \Psi(t) \rangle}{\int dt \langle \Psi(t) | \mathcal{O}_A | \Psi(t) \rangle}, \quad (29)$$

where  $\mathcal{O}_{A \rightarrow C}$  is the operator projecting onto states that satisfy  $A$  and evolve into  $C$ . This corresponds to Eq. (10) in the semi-classical picture. Note that the comment in footnote 12 still applies for the probabilities defined here, since the quantum evolution of  $|\Psi(t)\rangle$  is deterministic.

The probabilities in Eqs. (28) and (29) are well-defined. In particular, the numerators and denominators in these expressions are separately finite.<sup>24</sup> This finiteness can be understood as follows. Let us expand  $|\Psi(t)\rangle$  as

$$|\Psi(t)\rangle = a_1 |\Psi_1(t)\rangle + a_2 |\Psi_2(t)\rangle + \cdots, \quad (30)$$

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<sup>24</sup>Strictly speaking, they are finite only if  $|\Psi(t)\rangle$  is normalized. If not, they can be infinite, but these infinities cancel between the numerators and denominators, giving well-defined, finite probabilities.

where  $|\Psi_n(t)\rangle$  ( $n = 1, 2, \dots$ ) corresponds to a component which encounters  $A$ -satisfying past light cones  $n$  times in the cosmic history. The quantity in the denominators in Eqs. (28) and (29) is then given by

$$\int dt \langle \Psi(t) | \mathcal{O}_A | \Psi(t) \rangle = \sum_{n=1}^{\infty} n |a_n|^2. \quad (31)$$

Now, starting from any generic initial conditions, the probability of finding a past light cone that satisfies  $A$  is exponentially small (since it requires the geodesic to tunnel into a particular vacuum, or vacua, in which  $A$  can be satisfied). This implies that  $a_n$  scales as

$$a_n \sim e^{-cn} \quad (c > 0), \quad (32)$$

so that the sum in Eq. (31) converges. The expression in Eq. (31) is thus finite, as long as the multiverse state is appropriately normalized, e.g.  $\langle \Psi(t) | \Psi(t) \rangle = 1$ . The finiteness of other quantities can also be understood in a similar way.

The probabilities defined here are also “gauge invariant,” i.e. they do not depend on time parametrization. This is fairly obvious from the expressions in Eqs. (28) and (29), but it can also be understood along the lines above. Basically, our probabilities count the number of times each history,  $|\Psi_n(t)\rangle$ , encounters a particular past light cone(s). Since the coefficients  $a_n$  do not depend on time parametrization, the resulting probabilities do not depend on it either.

As discussed in Section 4.4, the complete evolution of  $|\Psi(t)\rangle$  requires the knowledge of quantum gravity, so that the probabilities in Eqs. (28) and (29) can only serve the role of defining the framework. To do a “practical” calculation, we need to focus on bulk physics, i.e.  $\rho_{\text{bulk}}(t)$  introduced in Eq. (23) (at the cost of unitarity in processes involving horizons). The observational conditions, such as  $A$ , should then be imposed only on the bulk part of the state. This is not a strong restriction, since our observational data are always on bulk physics in practice. The relevant projection operator is given by

$$\mathcal{O}_{\text{bulk},A} = \sum_i |\Psi_{\text{bulk},A,i}\rangle \langle \Psi_{\text{bulk},A,i}|, \quad (33)$$

where  $|\Psi_{\text{bulk},A,i}\rangle$  is an orthonormal set of the bulk part of the states satisfying condition  $A$ ; see Eq. (21). The probability  $P(B|A)$  is then given by

$$P(B|A) = \frac{\int dt \text{Tr} [\rho_{\text{bulk}}(t) \mathcal{O}_{\text{bulk},A \cap B}]}{\int dt \text{Tr} [\rho_{\text{bulk}}(t) \mathcal{O}_{\text{bulk},A}]}, \quad (34)$$

where the trace is over the bulk part of Hilbert space. The probability  $P(C|A)$  is defined similarly, with the replacement  $\mathcal{O}_{\text{bulk},A \cap B} \rightarrow \mathcal{O}_{\text{bulk},A \rightarrow C}$ .

The definition of Eq. (34) allows us to calculate probabilities without knowing quantum gravity. In principle it allows us to answer any questions, except for the ones regarding information stored

on a horizon at some time in the history. In Appendix B.2, we give sample calculations in toy landscapes, where it is shown that the results agree with those obtained using the semi-classical definition. Of course, to have definite numbers, an initial condition for  $\rho_{\text{bulk}}(t)$  needs to be specified. The issue of initial conditions will be discussed in Sections 5 and 8.

We finally mention the case where the multiverse is in a mixed state  $\rho(t)$ . In this case, the probability is given by

$$P(B|A) = \frac{\int dt \text{Tr} [\rho(t) \mathcal{O}_{A \cap B}]}{\int dt \text{Tr} [\rho(t) \mathcal{O}_A]}, \quad (35)$$

where the trace and projection operators act on the entire Hilbert space. The probability  $P(C|A)$  can be defined similarly.

## 4.6 Quantum-to-classical transition

As we have seen, our framework is (necessarily) quantum mechanical. On the other hand, our daily experience is certainly (almost) classical. How does this dichotomy arise? Why do we not observe, e.g., a state which is a superposition of different macroscopic configurations?

To be specific, let us suppose that the multiverse was in some highly symmetric state  $|\Psi(t_0)\rangle$  at an early moment. In particular, we consider that  $|\Psi(t_0)\rangle$  respects rotational symmetry. This leads to a question: why do we not observe a chair next to us in a rotationally invariant,  $s$ -wave state, i.e., a superposition of chairs with all different orientations? One might think that various physical processes, such as bubble nucleations, spontaneously break rotational symmetry. This is, however, too naive. The resulting state will still be a linear combination of states with various bubbles nucleating in all different locations such that rotational invariance is respected. Note that this problem is particularly acute in our context. With  $|\Psi\rangle$  being the quantum state for the entire multiverse, there is no “environment” with which  $|\Psi\rangle$  interacts to feel violation of rotational invariance.

Let us focus on a particular macroscopic object, such as a chair, desk, or human. As discussed above, we expect it to be in a rotationally invariant state:

$$|\Psi\rangle \sim \left( \left| \begin{array}{c} \text{chair} \\ \uparrow \end{array} \right\rangle + \left| \begin{array}{c} \text{chair} \\ \downarrow \end{array} \right\rangle + \dots \right), \quad (36)$$

where we have used a chair as an example and displayed explicitly only two configurations (upward and downward).<sup>25</sup> Now, consider a second object, say a human, next to the chair. Is the quantum state a direct product of the rotationally invariant chair state and the rotationally invariant human state? Namely,

$$|\Psi\rangle \stackrel{?}{\sim} \left( \left| \begin{array}{c} \text{chair} \\ \uparrow \end{array} \right\rangle + \left| \begin{array}{c} \text{chair} \\ \downarrow \end{array} \right\rangle + \dots \right) \otimes \left( \left| \begin{array}{c} \text{human} \\ \uparrow \end{array} \right\rangle + \left| \begin{array}{c} \text{human} \\ \downarrow \end{array} \right\rangle + \dots \right). \quad (37)$$

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<sup>25</sup>Since the chair is a macroscopic object,  $\left| \begin{array}{c} \text{chair} \\ \uparrow \end{array} \right\rangle$  ( $\left| \begin{array}{c} \text{chair} \\ \downarrow \end{array} \right\rangle$ ) itself can be an arbitrary superposition of microscopic states that macroscopically look like an upward (downward) chair.

The answer is no. To see this explicitly, consider the Hamiltonian for the chair

$$\left( \begin{array}{cc} \langle \text{chair} | & \langle \text{chair} | \end{array} \right) \hat{H} \left( \begin{array}{c} | \text{chair} \rangle \\ | \text{chair} \rangle \end{array} \right) = \left( \begin{array}{cc} A & B \\ B & A \end{array} \right), \quad (38)$$

where we have reduced the system to two states for presentation purposes, and the particular form of the matrix in the right-hand side is dictated by “rotational” symmetry ( $Z_2$  in the reduced system). We find that the two eigenstates of the Hamiltonian are  $(| \text{chair} \rangle \pm | \text{chair} \rangle)/\sqrt{2}$  with eigenvalues  $A \pm B$ , which are also eigenstates of “rotational” symmetry. However, for a macroscopic object,  $B$  is exponentially small, since it involves quantum tunneling between the upward and downward configurations. Consequently, the two states are nearly degenerate, and the preferred basis for the chair is determined by any small “rotational”-symmetry violating perturbations [31]. In the above chair-human system, the perturbation is the existence of the human, so that the state of the entire system is *not* given by Eq. (37), but by

$$|\Psi\rangle \sim \left( | \text{chair} \rangle \otimes | \text{human} \rangle + | \text{chair} \rangle \otimes | \text{human} \rangle + \dots \right), \quad (39)$$

where we have arbitrarily assumed that the dynamics prefers to align the orientations of the chair and human, rather than anti-align. (Our conclusion does not depend on this particular choice.) The process transforming a (macroscopic) composite system to an entangled form, e.g. as in Eq. (39), is called decoherence, which typically occurs with extremely short timescales [32].

We now see why we do not observe a superposition of different macroscopic configurations in our daily life. Equation (39) says that the chair always has a definite orientation *with respect to* the human, which we may identify with ourselves. We do not observe a superposition of chairs, which would have been possible if the state contained a term such as  $(| \text{chair} \rangle + | \text{chair} \rangle) \otimes | \text{human} \rangle$  as in Eq. (37). Note that quantum interferences between different terms in Eq. (39) are extremely small, since overlaps between macroscopically different configurations, such as  $| \text{human} \rangle$  and  $| \text{human} \rangle$ , are suppressed by the huge dimensionality of the corresponding Hilbert space. In fact, for any observables constructed out of local operators, matrix elements between macroscopically distinct states are highly suppressed, e.g.  $\langle \text{human} | \mathcal{O}_A | \text{human} \rangle \ll \langle \text{human} | \mathcal{O}_A | \text{human} \rangle, \langle \text{human} | \mathcal{O}_A | \text{human} \rangle$ . This, therefore, provides preferred bases for any macroscopic systems.

Of course, the *entire* state in Eq. (39) is still rotationally invariant. This, however, does not matter. Since we ourselves are a part of the state, we never observe the rotationally invariant state  $|\Psi\rangle$ . In fact, rotational invariance of the state  $|\Psi\rangle$  implies that for any component  $|\psi\rangle$  in  $|\Psi\rangle$ , there are terms whose *entire histories* are related to  $|\psi\rangle$  via rotational symmetry (see e.g. the first two terms in Eq. (39)). For a macroscopic system, the evolution of these terms are mutually independent for all practical purposes. Thus, we may simply use  $|\psi\rangle$  (or any other term related to

it by the symmetry) when making predictions. Of course, we may still use the entire state  $|\Psi\rangle$  if we want—the two procedures give identical results.

The consideration here addresses various questions associated with the quantum-to-classical transition, for example, why spontaneous symmetry breaking occurs at all [31], why density perturbation in inflation becomes classical [33], and why we do not observe a superposition of bubble universes. In our context, the multiverse state is indeed a superposition of various macroscopically different configurations. We see (almost) classical physics because we—who are *a part of* the state—are correlated (entangled) with the rest of the multiverse. Technically, our framework incorporates quantum measurement processes in the form of the imposition of observational conditions, such as  $A$ ,  $B$ , and  $C$  in Eqs. (28) and (29). This procedure exactly extracts information encoded in the correlations between ourselves and the rest of the world.

## 4.7 The possibility of a reduced Hilbert space

We finally mention the possibility that the Hilbert space of the theory is actually smaller than that in Eq. (21).

Consider an arbitrary semi-classical configuration of spacetime  $\mathcal{M}$ . In the multiverse, such a configuration arises as components of  $|\Psi(t)\rangle$  at various times  $t_a$  ( $a = 1, 2, \dots$ ). Let us define an ensemble of all these states:

$$\mathcal{E} = \{\mathcal{O}_{\mathcal{M}}|\Psi(t_a)\rangle\}, \quad (40)$$

where the states are at fixed times  $t_a$ , and  $\mathcal{O}_{\mathcal{M}}$  is the projection operator. Since a component of a pure state  $|\Psi(t)\rangle$  is also a pure state, elements of  $\mathcal{E}$  are all pure states, which take the form

$$|\Psi_i\rangle = |\Psi_{\text{bulk},i}\rangle \otimes |\Psi_{\text{horizon},i}\rangle, \quad (41)$$

where  $i = 1, \dots, \dim(\mathcal{E})$ .

Now, the holographic principle says that (the logarithm of) the number of *all possible* bulk configurations is bounded by the horizon area in Planck units:

$$S_{\mathcal{M},\text{bulk}} \equiv \ln \{\dim(\cup_i |\Psi_{\text{bulk},i}\rangle)\} \leq \frac{\mathcal{A}_{\text{horizon}}}{4l_P^2}. \quad (42)$$

We expect that this inequality is saturated, since the multiverse realizes all possible states throughout the history. It is also reasonable to assume that the horizon can contain only  $O(1)$  bits of information per Planck area. Suppose that this number takes a special value of  $1/4$ , i.e.  $S_{\mathcal{M},\text{horizon}} \equiv \ln \{\dim(\cup_i |\Psi_{\text{horizon},i}\rangle)\} = \mathcal{A}_{\text{horizon}}/4l_P^2$ . Then we find

$$S_{\mathcal{M},\text{bulk}} = S_{\mathcal{M},\text{horizon}}, \quad (43)$$

i.e. the numbers of bulk and horizon degrees of freedom are the same for a fixed  $\mathcal{M}$ .

If Eq. (43) is true for arbitrary  $\mathcal{M}$ , then it allows for an interesting possibility that the Hilbert space of the theory is actually the square root of Eq. (21), namely, there are one-to-one correspondences between elements of  $\mathcal{H}_{\mathcal{M},\text{bulk}}$  and  $\mathcal{H}_{\mathcal{M},\text{horizon}}$  for all  $\mathcal{M}$ . If this is the case, then we can provide a *complete* description of the multiverse in terms of *either*  $|\Psi_{\text{bulk}}(t)\rangle$  *or*  $|\Psi_{\text{horizon}}(t)\rangle$ .

## 5 Initial Conditions

The framework developed so far allows us to make predictions/postdictions once initial conditions are given. On the other hand, the framework itself does not provide a unique initial condition. This is, in fact, as it should be. Remember that the measure problem of eternal inflation has *a priori* nothing to do with the “beginning” of spacetime. Once eternal inflation occurs in the regime where semi-classical analyses are valid, then it already leads to the predictivity crisis—any event that can happen will happen infinitely many times. Our general framework should be able to regulate these infinities, starting from *any* eternally inflating vacuum. Namely, the framework should be sufficiently modular such that any of such vacua can be used as initial conditions, e.g., on  $\Sigma$ .

On the other hand, the framework also provides a useful tool to probe the real beginning of spacetime, i.e., the initial condition of the multiverse. In this section, we discuss the issue of initial conditions from both these two perspectives. We first consider what initial conditions we need to use to calculate probabilities starting from an arbitrary eternally inflating state. We then briefly discuss the beginning of spacetime, deferring full discussions to Section 8.

### 5.1 Semi-classical predictions from eternal inflation

Suppose we consider a spacetime region that is eternally inflating, with the fields taking values collectively denoted as  $\phi$ . We want to derive predictions for the future, e.g.  $P(B|A)$  and  $P(C|A)$ , using our framework. What initial condition should we impose on the multiverse state?

We first note that the quantum state of the universe is *not* uniquely determined by saying that the universe is in a (approximately) de Sitter state with field values  $\phi$ . There are two issues associated with this. The first comes from the fact that the de Sitter space has a nontrivial entropy  $S = 3/8G_N^2V(\phi)$ , which implies that this space, in fact, represents a statistical ensemble of many quantum states with  $S$  degrees of freedom. Here,  $G_N$  is Newton’s constant and  $V(\phi)$  is the potential energy density. The only thing we know is that the system is in a thermal state with de Sitter temperature  $T = \sqrt{2G_NV(\phi)}/3\pi$  at the level of semi-classical approximation. In our framework, such a thermal picture arises (only) after integrating out horizon degrees of freedom, assuming complete ignorance about these degrees of freedom. This implies that we can (only) use the formalism based on the bulk density matrix,  $\rho_{\text{bulk}}(t)$ , when we address the questions discussed

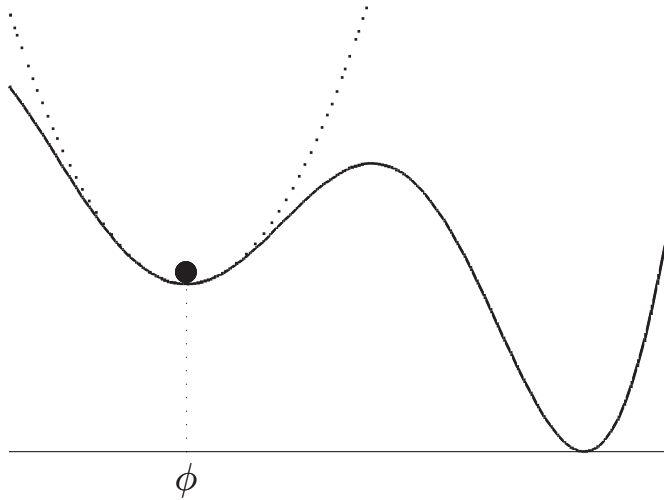


Figure 6: The Hamiltonian  $\hat{H}_\phi$  is that of a theory with the potential having the global minimum at  $\phi$  and mimicking the original theory around  $\phi$ . The potential of this theory and that of the original theory are drawn by the dotted and solid lines, respectively.

here.

The second issue is that eternally inflating spacetime is in fact not pure de Sitter—it has a nonzero decay width. This implies that we need to specify an initial hypersurface on which the universe was still in an eternally inflating phase. This specification affects physical predictions at late times, albeit weakly [14, 15]; namely, different choices for the hypersurface correspond to different physical setups. In our framework, a canonical choice for equal time hypersurfaces is to take them to be the observer’s past light cones bounded by the stretched horizon. Therefore, it is natural to define an initial condition on one of these hypersurfaces. Indeed, this choice is physically well motivated, since the spacetime region the observer can actually see is limited to that within his/her past light cone. With this choice, we are specifying that the entire universe was in an eternally inflating phase at some point in the history, *as seen by* the observer.

The initial condition we need to use, then, is that the universe is in a (approximately) de Sitter state at some initial moment, say at  $t = t_0$ :

$$\rho_{\text{bulk}}(t_0) \simeq \frac{1}{\text{Tr } e^{-\beta \hat{H}_\phi}} e^{-\beta \hat{H}_\phi}, \quad (44)$$

where  $\beta = 1/T = \sqrt{3\pi/2G_N V(\phi)}$ , which depends on  $\phi$ , and  $\hat{H}_\phi$  is the Hamiltonian operator defined for a theory which mimics the original theory around  $\phi$  but has the stable vacuum at  $\phi$  (see Fig. 6). (The trace in the denominator is taken in the Hilbert space of this theory.) There is, of course, an ambiguity in defining such a theory, and thus  $\hat{H}_\phi$ , since there is no guideline for choosing

a potential of the new theory above the potential barrier of the original theory. This ambiguity, however, leads to only exponentially suppressed corrections in the final results, as long as the de Sitter temperature  $T$  is much smaller than the barrier height, which is necessary anyway for the minimum  $\phi$  to be meta-stable. (And we do not expect semi-classical calculations to correctly reproduce exponentially suppressed contributions anyway.)

With the initial condition of Eq. (44), we can now derive any predictions of eternal inflation using Eq. (34). As discussed before, this can be done without knowing quantum gravity.

## 5.2 Issues with the initial condition of the multiverse

Let us now consider the beginning of spacetime. This will require a theory of initial conditions beyond what we have developed so far. Here we only discuss several issues associated with it, leaving more complete discussions to Section 8.

One possibility for the beginning is that it is purely determined by semi-classical considerations. Suppose that spacetime starts from some highly symmetric state, with the fields sitting at an extremum of the potential. Let  $\phi_a$  ( $a = 1, \dots, n$ ) denote  $n$  such extrema that have positive energy densities. We may then consider the initial bulk density matrix

$$\rho_{\text{bulk}}(t_0) \simeq \sum_{a=1}^n \lambda_a \frac{1}{\text{Tr } e^{-\beta_a \hat{H}_{\phi_a}}} e^{-\beta_a \hat{H}_{\phi_a}}, \quad (45)$$

where  $\sum_{a=1}^n \lambda_a = 1$ ,  $\beta_a = \sqrt{3\pi/2G_N V(\phi_a)}$ , and  $\hat{H}_{\phi_a}$  is the Hamiltonian defined around the extremum at  $\phi_a$ .<sup>26</sup> For

$$\lambda_a \propto \exp\left(\pm \frac{3}{8G_N^2 V(\phi_a)}\right), \quad (46)$$

this corresponds to Hartle-Hawking [34] (the upper sign) and tunneling [35] (the lower sign) proposals.<sup>27</sup> There are some unsatisfactory features in these “semi-classical beginning” pictures (although they may, in some sense, be unavoidable; see discussion in Section 8.4). First, the fact that the initial condition specifies only  $\rho_{\text{bulk}}$ , and not  $|\Psi\rangle$ , means that there is an *intrinsic uncertainty* which we cannot hope to reduce; in particular, we in principle cannot predict exact states for certain (horizon) degrees of freedom. Second, the expression of Eq. (45) is only approximate, as the definition of  $\hat{H}_{\phi_a}$  has an ambiguity, and we do not see any obvious way to make it exact.

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<sup>26</sup>This equation may also apply if the landscape is reducible, i.e., if there are multiple sectors in the potential that are not mutually connected by any physical processes.

<sup>27</sup>The Hartle-Hawking probability can be understood not as a probability of creating universes, but as a thermal distribution of universes in a theory with a de Sitter ground state [36]. In addition, this probability has the serious problem of overwhelming Boltzmann brain observers [37]. Therefore, the tunneling probability seems (relatively) more suitable in the present context.

An alternative possibility for the beginning is that the theory of initial conditions determines the multiverse to be in a specific pure state  $|\Psi(t_0)\rangle$  at the earliest moment. According to our hypothesis, this implies that the multiverse is in a pure state for arbitrary time  $t$ ,  $|\Psi(t)\rangle$ .

In either of these cases, if we admit the existence of *the* initial state—which is the case in any scenario along the lines of creation from “nothing” [38]—then the quantum observer principle is necessarily violated there, since we cannot evolve the state further back. While this is possible, in Section 8 we will explore the possibility that the principle is in fact not violated throughout the whole history, which will suggest that our multiverse is a “fluctuation” in some larger structure.

## 6 Quantum Measurements and Global Spacetime

In this section, we see that our framework allows for a unified treatment of quantum measurement processes and the eternally inflating multiverse. We conclude that the eternally inflating multiverse is the same as many worlds in quantum mechanics. We also discuss the relation of our single observer picture to the conventional, global spacetime picture.<sup>28</sup>

### 6.1 Unification of the multiverse and many worlds in quantum mechanics

So far, we have mainly focused on issues at very large scales, e.g. bubble universes in eternal inflation, in applying our formalism. This, however, need not be the case. In fact, the formalism applies equally to any (even microscopic) quantum processes without modification.

Suppose the multiverse starts from a highly symmetric state  $|\Psi(t_0)\rangle$ . This state evolves into a superposition of states in which various bubble universes nucleate in various spacetime locations. As time passes, a state representing each universe further evolves into a superposition of states representing various possible cosmic histories, including different outcomes of “experiments” performed within that universe. (These “experiments” may, but need not, be scientific experiments—they can be any physical processes.) At late times, the multiverse state  $|\Psi(t)\rangle$  will thus contain an enormous number of terms, each of which represents a possible world that may arise from  $|\Psi(t_0)\rangle$  consistently with the laws of physics. A schematic picture of these “branching” processes is given in Fig. 7.

The resulting picture is remarkably simple. From the initial state  $|\Psi(t_0)\rangle$ , the multiverse simply evolves deterministically according to the quantum mechanical evolution law. This evolution, however, is not along an axis in Hilbert space that is determined by operators local in spacetime. Therefore, at late times, the multiverse state is misaligned with (an enormous number of) axes determined by the local operators. (These axes are analogues of Fock states in usual quantum field

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<sup>28</sup>A similar relation is discussed independently by Raphael Bousso in the context of geometric cutoff measures [9].

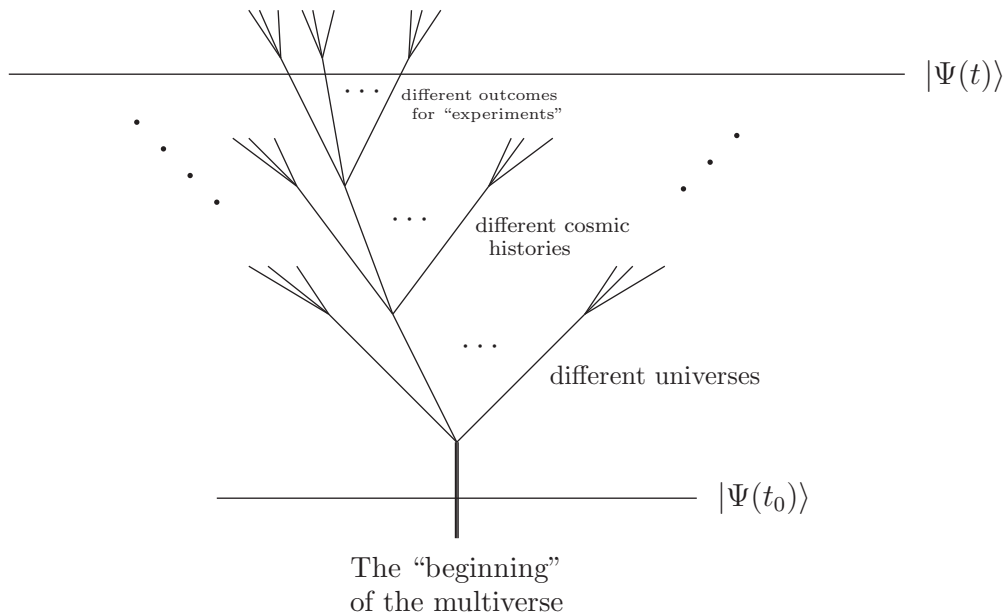


Figure 7: A schematic picture for the evolution of the multiverse state  $|\Psi(t)\rangle$ . The state is described from the viewpoint of a single “observer” (geodesic) traveling the multiverse.

theories.) This makes the multiverse state a superposition of a huge number of terms corresponding to different cosmic histories:

$$|\Psi(t)\rangle \approx \sum_i |\text{possible world } i \text{ at time } t\rangle, \quad (47)$$

when expanded in the basis determined by local operators. As discussed before, all these histories are described from the viewpoint of a single “observer” (geodesic).

The picture given above is precisely that of the many-worlds interpretation of quantum mechanics [39]. Therefore, we conclude that *the multiverse is the same as (or a specific manifestation of) many worlds in quantum mechanics*. In fact, when we ask physical questions, we “inject” these questions to the theory by imposing conditions such as  $A$ ,  $B$ , and  $C$  in Eqs. (28) and (29). The questions may be about global properties of the universe, or about outcomes of a specific experiment being performed. Our framework, therefore, provides a fully unified treatment of (even microscopic) experiments and the multiverse. Indeed, it applies to any physical processes from the smallest (the Planck length) to the largest (the apparent horizon) scales.

Incidentally, if we are interested only in the future of a particular macroscopic configuration at time  $t$ , then we may drop all the terms in Eq. (47) except for ones corresponding to the specified configuration. This is because the superposition principle guarantees that the evolution

of the retained terms is independent of the dropped terms, and matrix elements of macroscopic observables between states with different macroscopic configurations are highly suppressed (see Section 4.6) so the dropped terms do not affect future predictions. *This operation of truncating the state is precisely what is called wavefunction collapse*, which is indeed an extremely good approximation when we ask questions about a system with large degrees of freedom.<sup>29</sup> (Note that a system here includes an experimental apparatus, in addition to the object being measured.) It is, however, not an exact procedure in general, nor necessary for making predictions.

## 6.2 Connection to (“reconstruction” of) the global picture

What is the relation of our picture based on a single quantum observer to the picture based on global spacetime? One (extreme) attitude is to consider that “physical reality” is simply the multiverse quantum state, so that no other picture is needed. This indeed makes sense because all physical questions (regarding predictions/postdictions) can be answered using that state, following the prescription in Section 4. On the other hand, it is useful to understand a connection between the present picture and more conventional, global spacetime picture. Here we study this issue.

One way of developing intuition about the connection is to consider a large (infinite) number of identical multiverse states. As shown in conventional analyses of frequency operators [40], the resulting product state can be reorganized as

$$|\Psi(t)\rangle^{\otimes N} = \left( \sum_i c_i |\alpha_i\rangle \right)^{\otimes N} \xrightarrow{N \rightarrow \infty} \propto \left( |\alpha_1\rangle^{|\alpha_1|^2 N} \otimes |\alpha_2\rangle^{|\alpha_2|^2 N} \otimes \dots \right) + \text{permutations}, \quad (48)$$

up to a zero-norm part of the state. Along the lines of Ref. [41], we may interpret the rightmost expression to represent each outcome  $|\alpha_i\rangle$  spreading over (global) spacetime. In particular, any experimental results may be viewed as distributed over the multiverse, allowing us to interpret outcomes of a quantum measurement in the frequentist’s sense over spacetime.<sup>30</sup>

Let us now study the connection between the two pictures in more detail. (The following analysis does not require Eq. (48), which was presented simply to help developing intuition.) Remember that the multiverse state  $|\Psi(t)\rangle$  contains all possible worlds that can consistently arise in the single observer’s viewpoint. For example, in a spatially homogeneous region, an event that can occur may occur anywhere in space, so the multiverse state contains a superposition of terms in which the event occurs in all different locations, as schematically illustrated in the upper line of Fig. 8. Now, in the global spacetime picture, these terms can be interpreted to correspond to the

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<sup>29</sup>The approximation may even be “exact” if the system is large enough that the effect from the dropped terms is smaller than what is measurable within the limitation of the uncertainty principle.

<sup>30</sup>In contrast to Ref. [41], in which the assumption of statistical uniformity played an important role, our argument here requires only Eq. (48), since we do not have the measure problem. In fact,  $N$   $|\Psi(t)\rangle$ ’s in Eq. (48) are *identical* copies by construction.

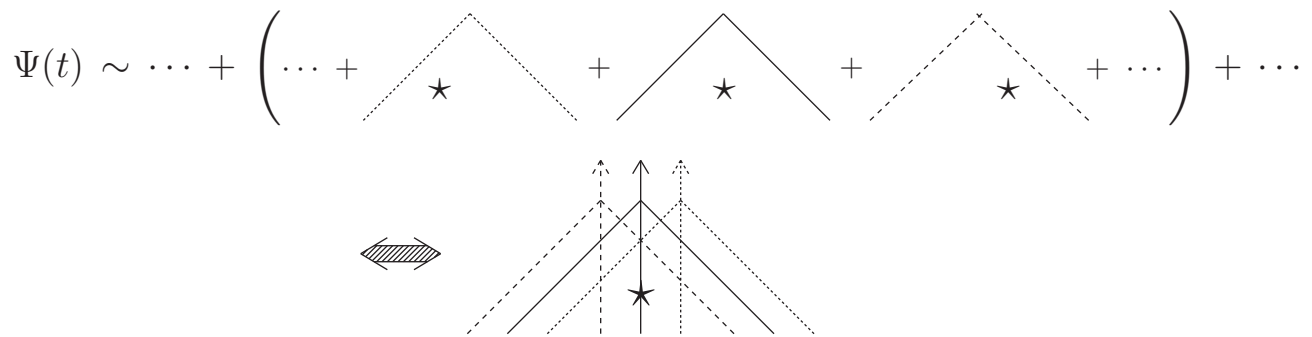


Figure 8: A schematic depiction of the relation between the single quantum observer picture (the upper line) and the global spacetime picture (the lower line). The location of an event, represented by stars, is discretized for illustrative purposes.

*same* spacetime region as viewed from *different observers (geodesics)*, as illustrated in the lower line of the figure.

In general, space is not homogeneous, i.e., the values of the coefficients are not identical for terms that have the event occurring in different locations relative to some other object (since the existence of the object breaks the symmetry). In the global spacetime picture, this means that the “density” of observers (geodesics) are not uniform in such regions. Note that the picture that spacetime is scanned/penetrated by a large number of geodesics is precisely what we had in Section 3. In fact, *(semi-classical) global spacetime can always be “reconstructed” from the single quantum observer picture* in the manner described here.

## 7 (No) Problems associated with Geometric Cutoffs

In a conventional treatment of the multiverse, one introduces a geometric cutoff to regularize infinite spacetime. The (relative) probabilities are then defined by counting the number of events within the cutoff, which is eliminated at the end of the calculation either by sending it to infinity or through a certain averaging procedure [5]. This treatment, however, introduces an arbitrariness associated with the choice of the cutoff, and also leads to peculiar conclusions such as the “end” of time [12]. Here we show that our framework does not suffer from these difficulties.

### 7.1 Youngness paradox, Boltzmann brains, and other problems

Arguably, the earliest, and numerically most severe, problem encountered in the context of eternal inflation is the youngness paradox [42]. Suppose we take FRW time while the universe is still in

the eternally inflating phase, and then extend this time coordinate to future keeping a synchronous gauge condition. This defines a global time coordinate over the entire (semi-classical) multiverse. We can now define the relative probability of two classes of events by the ratio of the numbers of events occurring before a fixed time  $t_c$ , taking  $t_c \rightarrow \infty$  at the end of the calculation [43].

This most naive—and seemingly innocuous—procedure, however, leads to violent contradictions with observations. Since the spatial volume of the eternally inflating region is growing exponentially with time, the rate at which bubble universes form is also increasing exponentially, proportional to the volume. This implies that, at any given time, there are enormously more younger universes than older ones. For instance, the number of universes with  $T_{\text{CMB}} \simeq 3$  K is a factor of  $\sim 10^{10^{59}}$  larger than that with  $T_{\text{CMB}} = T_0 \simeq 2.725$  K. It is hard to imagine that such an immense bias towards younger universes is compensated by any possible anthropic weight factor.

Our framework does not suffer from this problem. This is because the increase of spatial volume is not rewarded in calculating probabilities. (This is also true in classes of geometric cutoff measures proposed in Refs. [44, 45].) To see this, let us consider the quantum probabilities defined in Section 4. In this picture, the probabilities are calculated from the viewpoint of a single observer, so it is rather evident that the volume increase is not rewarded. For instance, there is obviously no gain in probabilities by staying longer in an eternally inflating phase. The same can also be seen in the semi-classical picture—spacetime expansion increases spatial volume, but only at the cost of diluting observers (geodesics). Since there is no reward for volume increase, any problems associated with volume weighting (such as  $Q$ -catastrophe [46]) do not arise in our framework.

The present framework is also free from ambiguities associated with the question “what is an observer?”;<sup>31</sup> e.g., does an observer mean a civilization, an individual, or some sort of consciousness? (Even what about a dog seeing a tree?) In fact, the answer to it should be specified *already in a question we ask*, i.e. in prior condition  $A$ . For example, we can ask the probability of an individual (defined carefully) to see a certain event, a civilization (again, carefully defined) to obtain a certain result from a certain experiment, and so on—as long as the question is well defined, we will obtain a well defined answer. We also mention that our framework does not suffer from the ambiguity for defining probabilities which arises if an observer can condition only a part of a wavefunction, as occurs in a global (quantum) description of the universe [47]. Our framework avoids this problem because the state is defined as viewed from a single observer, so that the conditioning (such as  $A$ ,  $B$ , and  $C$ ) is on the *entire* multiverse state, through Eqs. (28) and (29). In fact, these equations are nothing but the standard Born rule, which plays an essential role in our framework.

The Boltzmann brain problem is the statement that if (a part of) the universe stays in a (approximately) de Sitter phase too long, then ordinary observations made by ordinary observers are

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<sup>31</sup>So far, we have been using the word “observer” to simply mean a geodesic in the multiverse. In contrast, the observer here means a “real observer” (called an experimenter in Section 4.2), who performs experiments/observations and collects data. I hope there is no confusion.

completely overwhelmed by disordered “observations” made by vacuum thermal fluctuations [48].<sup>32</sup> For example, if the naive synchronous time cutoff is adopted (as in the first paragraph of this subsection), then this leads to the peculiar conclusion that our universe must decay within  $\approx 20$  billion years. In fact, this conclusion does not even require the existence of other universes. The problem is generically worse if we take into account other universes.

Our framework avoids this problem under rather mild assumptions about the vacuum structure of the landscape. The situation is similar to those in classes of geometric cutoffs that do not reward volume increase [49]. Let us first formulate the problem precisely in the current framework. Suppose that some “observation” is made. This requires us to take prior condition  $A$  to be classes of information processing occurring in spacetime in some physical form (which includes firings of neural signals in a human brain). We then take condition  $B$  to be that these “observations” find some ordered pattern (e.g. the sight “seen” by this process is an ordered world obeying regular rules). The Boltzmann brain problem arises if we obtain  $P(B|A) \lll 1$ . Since the world we (or I/you) see is ordered, such a result would contradict observations.

The probability of a random thermal fluctuation to compose an “observation” is suppressed by a huge Boltzmann factor associated with a (macroscopic) configuration corresponding to the observation—estimates for this factor span the range  $\approx \exp(10^{O(10-100)})$  [49]. Since our framework does not reward volume increase, the problem is avoided if de Sitter vacua decay before Boltzmann brains start dominating. This gives only weak constraints on lifetimes of de Sitter vacua:  $\tau_{\text{dS}} \lesssim \exp(10^{O(10-100)})$ , where the exponent depends on a vacuum.<sup>33</sup> Indeed, the fact that de Sitter vacua decay within such timescales is consistent with what we know about string theory [50]. The resulting lifetimes are also sufficiently short to avoid Poincaré recurrences [51].

## 7.2 The “end” of time

We now discuss the issue of the “end” of time. It has recently been pointed out that *any* simple geometric cutoffs lead to the peculiar conclusion that time should “end” [12]. Suppose we consider two classes of events: an experimenter sees (i) 1 o’clock and (ii) 2 o’clock on his/her watch. Since there are always experimenters for whom the observation of 1 o’clock occurs before the cutoff while that of 2 o’clock after, the numbers of observations of 1 and 2 o’clocks,  $N_{1,2}$ , satisfy  $N_1 > N_2$ . An important point is that, since events in eternally inflating spacetime are dominated by a late-time attractor regime, the effect of the cutoff does *not* decouple when it is sent to infinity, i.e.,

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<sup>32</sup>If the multiverse is in a pure state, then a particular de Sitter vacuum does not have thermal fluctuations because it corresponds to a single quantum state. We can, however, consider an ensemble of de Sitter vacua throughout the multiverse which macroscopically look the same. We then find that the standard argument for Boltzmann brains still applies to this ensemble; namely, the problem may exist even in the case where the multiverse is in a pure state.

<sup>33</sup>Units of time do not matter here, since the right-hand side is so large. Also, if a vacuum does not support Boltzmann brains, then that vacuum need not obey this bound.

$P_{\text{end}} \equiv 1 - N_2/N_1 \rightarrow 0$ . This implies that there is a nonzero probability that an experimenter who saw 1 o'clock never sees 2 o'clock, *even if* the watch or experimenter does not break or die in between—for some experimenters, time simply “ends.”

Our framework does not lead to this conclusion. Consider the probability that an experimenter who saw 1 o'clock will see 2 o'clock. This is given by Eq. (28) (or Eq. (9)), taking  $A$  to be an experimenter seeing 1 o'clock, while  $B$  the same experimenter seeing 2 o'clock. It is then obvious that, since the time evolution of the system follows standard physical laws, we should find

$$P(2 \text{ o'clock} | 1 \text{ o'clock}) = 1, \quad (49)$$

as long as the watch or experimenter does not break/die, which we are assuming here. Namely, time does *not* end in a way discussed in Ref. [12]. (Of course, it can still end in other ways, e.g. at spacetime singularities.) The ultimate reason behind this can be traced in a sentence in Ref. [12]: “In eternal inflation, however, one first picks a causal patch; then one looks for observers in it.” Our framework does not follow this approach. We instead pick an observer first, and then construct the relevant spacetime regions associated with it.

Let us now ask: what is *the* relative probability of the watch showing 1 o'clock and 2 o'clock? A natural way of defining the question is to take conditions  $A$  and  $B$  as: (A) A watch is located at (or has an overlap with) the tip of the past light cone of an observer (geodesic), and its long hand is on the twelve; (B) The short hand of the watch is on the (i) one or (ii) two. Then it is easy to see that, in an expanding universe, we obtain

$$\frac{P(\text{“2 o'clock”})}{P(\text{“1 o'clock”})} < 1, \quad (50)$$

as depicted in the left panel of Fig. 9. (This is related to the fact that our procedure can be viewed as a sort of time cutoff in expanding universes; see Appendix A.) How can this be consistent with Eq. (49)?<sup>34</sup>

The answer is that the probabilities  $P(\text{“1 o'clock”})$  in Eq. (50) and  $P(1 \text{ o'clock})$  in Eq. (49) (and similarly for 2 o'clock) are different quantities. Specifically, Eq. (49) should better be written more explicitly as

$$P(\text{The same experimenter sees 2 o'clock.} | \text{An experimenter sees 1 o'clock.}) = 1, \quad (51)$$

which can certainly be consistent with Eq. (50); see the right panel of Fig. 9.<sup>35</sup> The probability is “lost” in Eq. (50) not because time ended in between, but because some of the “observers” (geodesics) who “saw” 1 o'clock simply miss the watch at 2 o'clock because of spacetime expansion (Fig. 9, left).

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<sup>34</sup>I thank Raphael Bousso for bringing my attention to this issue.

<sup>35</sup>Strictly speaking, there can be a (small) possibility that the experimenter or watch leaves the causal horizon of the observer (geodesic), making the probability of Eq. (51) smaller than 1. This is, however, a usual physical process already existing in general relativity, and has nothing to do with the end of time.

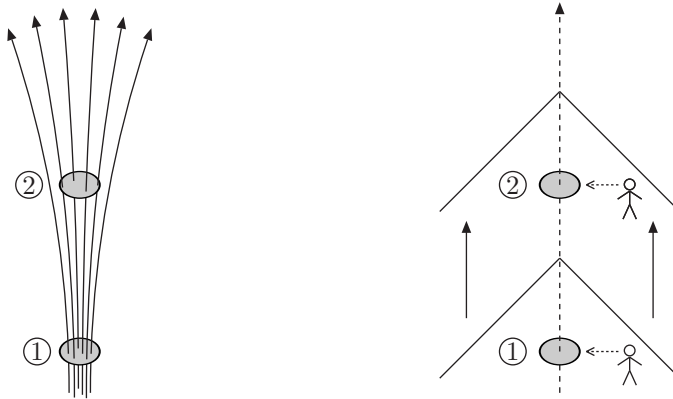


Figure 9: Some of the “observers” (geodesics) passing the watch at 1 o’clock do *not* pass it at 2 o’clock (left). On the other hand, an experimenter who saw the watch at 1 o’clock *will* see it at 2 o’clock (right). Here, the hats in the right panel represent past light cones.

## 8 Discussions

A predictivity crisis in eternal inflation has long been a major problem in cosmology. An important aspect of the problem comes from its robustness. Once we have a sufficiently meta-stable de Sitter vacuum, it produces an infinite number of events in an infinite number of lower energy vacua, making predictions impossible without an appropriate regularization. The high sensitivity of predictions on the regularization prescription has plagued many physicists over the last two decades. In fact, the problem has, a priori, nothing to do with the landscape or quantum gravity—it already exists in classical general relativity.

Like black hole physics, this predictivity problem has told us a lot about fundamental aspects of gravity and spacetime. The youngness paradox says that the measure based on a synchronous time cutoff in global spacetime does not work, despite the fact that it apparently seems most natural and innocuous. The Boltzmann brain problem implies that de Sitter vacua (at least ones that can support complexities required for “observations”) must be unstable, with lifetimes much shorter than their Poincaré recurrence times. Measures avoiding these problems, especially ones based on local pictures [45, 52], were put forward, but they still suffer from a peculiar conclusion that time should “end” even if there is no corresponding singularity in general relativity.

In this paper, we presented a framework which gives well-defined predictions and yet does not suffer from these difficulties. The framework is formulated consistently within a fully quantum mechanical treatment of the multiverse. (The semi-classical picture can be derived by “integrating out” physics associated with quantum gravity.) We argued that the *entire* multiverse is described

*purely* from the viewpoint of a single “observer.” A complete description of the physics is obtained in spacetime regions that the observer can causally access and are bounded by his/her apparent horizons. In conventional geometric cutoff measures, one first picks a spacetime region and then looks for observers in it. We do not follow this approach. We instead pick an observer first, and then construct the relevant spacetime regions associated with it.

The resulting picture is quite satisfactory. As viewed from a single observer, probabilities keep being “diluted” because of continuous branching of the state into different semi-classical possibilities, which is caused by the fact that the evolution of the multiverse state is not along an axis in Hilbert space determined by operators local in spacetime. This makes it possible to obtain well-defined predictions according to the standard Born rule. We may say that *it is quantum mechanics that solves the measure problem in eternal inflation*. Indeed, our framework allows for a completely unified treatment of quantum measurement processes and the multiverse. We conclude that *the eternally inflating multiverse and many worlds in quantum mechanics are the same*.

The multiverse state  $|\Psi(t)\rangle$ , or  $\rho(t)$ , is literally “everything” for making predictions. For example, even our own existence appears in components of  $|\Psi(t)\rangle$  at some time(s)  $t_j$ . Physical predictions can then be made by extracting correlations between ourselves (or our experimental apparatus) and the surroundings, using Eq. (28) or (29). There is no need to introduce anything beyond these basic elements—in particular, it is not necessary to introduce wavefunction collapse, environmental decoherence, or anything like those (although these concepts will still be useful when applied to understanding structures *inside*  $|\Psi(t)\rangle$ ). Indeed, there is no “external observer” that performs measurements on  $|\Psi(t)\rangle$ , and there is no “environment” with which  $|\Psi(t)\rangle$  interacts. The only task left for us is to find the “Hamiltonian” for quantum gravity,  $\hat{H}$ , and the boundary conditions determining the multiverse state, which, hopefully, a complete understanding of string theory would give us.

The picture described above provides a satisfactory answer to the question raised at the beginning of this paper: what is the meaning of the phrase “In an eternally inflating universe, anything that can happen will happen; in fact, it will happen an infinite number of times”? We can now say that anything that can happen will happen with a nonzero quantum mechanical probability. The same event can occur many times during the cosmic history, although the probability for that to happen is small. The volume increase in the semi-classical picture of eternal inflation ensures that all these events can look identical, but it does not mean that probabilities of multiple occurrences are larger. In fact, the probabilities are *not* obtained by simply *counting* the number of events in (regulated) semi-classical spacetime, which already assign (implicitly) equal probability weights for all these events. Infinities appear only if one asks a question an infinite number of times, which obviously does not affect the probabilities. Indeed, one can view that the probabilities are more fundamental, which exist regardless of whether one asks a question or not.

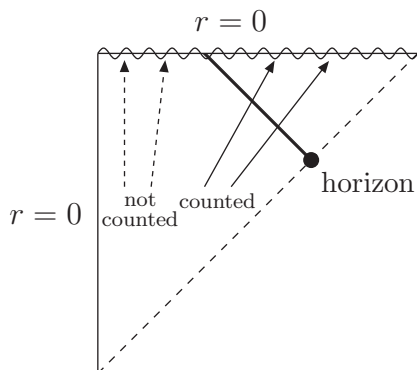


Figure 10: A Penrose diagram representing inside a black hole. The light sheet (thick solid line) associated with a horizon (dot) counts only degrees of freedom that pass the sheet before hitting the singularity.

Our framework has a number of implications beyond what have been discussed so far. Below, we consider some of them.

## 8.1 The ultimate fate of the multiverse

The evolution of the multiverse state  $|\Psi(t)\rangle$  is supposed to be unitary. On the other hand, some of the components in  $|\Psi(t)\rangle$  hit black hole or big crunch singularities at finite proper times. What happens to these components?

We conjecture that these components simply “disappear” from  $|\Psi(t)\rangle$ . This conjecture is motivated by the holographic principle, especially the covariant entropy bound [26]. Let us consider a black hole and count the entropy inside the horizon. The covariant entropy bound implies that degrees of freedom are counted only if they pass the light sheet before hitting the singularity; see Fig. 10. One may interpret this such that the light sheet is extended all the way to the center, but the degrees of freedom that hit the singularity—where the curvature is of order the Planck scale—“disappear” from the theory. In fact, this interpretation is consistent with what is suggested by black hole physics: there are essentially no degrees of freedom behind the stretched horizon where the local temperature is above the Planck scale. In our context, this implies that a component that hits the singularity should be eliminated from  $|\Psi(t)\rangle$  (i.e. the coefficient of the term should be set to zero) at the time of the hitting. The information about the disappearing component remains in other components in  $|\Psi(t)\rangle$ , e.g. in the form of Hawking radiation, so that no contradiction with reversibility of quantum evolution arises. Note that this argument would fail if there were exact global symmetries that prohibit degrees of freedom to disappear; but string theory suggests that

any global symmetries are inevitably broken by Planck scale physics [53].<sup>36</sup> The same argument as the one here also applies to big crunch singularities.

The conjecture given above has an important consequence for the ultimate fate of the multiverse. Starting from any initial state, components that hit singularities disappear from  $|\Psi(t)\rangle$ . Since observers (geodesics) inside anti de Sitter bubbles hit singularities within finite times, they do not remain in  $|\Psi(t)\rangle$  far in the future.<sup>37</sup> In addition, any observers in de Sitter bubbles or non-supersymmetric Minkowski bubbles eventually experience decays into lower energy vacua,<sup>38</sup> so they either disappear into singularities or end up in supersymmetric Minkowski vacua, which are stable due to the positive energy theorem [56]. Therefore, in the infinite future, the multiverse state becomes

$$|\Psi(t)\rangle \xrightarrow{t \rightarrow \infty} \sum_i |\text{Supersymmetric Minkowski world } i\rangle, \quad (52)$$

where the sum runs over states in vacua with varying low energy physics, e.g., matter content, spatial dimensions, and the amount of supersymmetries. This result has an important implication on a possible theory describing the entire history of the multiverse, as discussed in Section 8.2.

One interesting question is which vacua can exist in the sum in the right-hand side of Eq. (52). In the string landscape, not all possible supersymmetric theories may be realized with vanishing cosmological constant. For example, moduli might not all be stabilized supersymmetrically with vanishing superpotential, and the unfixed moduli might be pushed away by nonperturbative effects, excluding four dimensional  $\mathcal{N} = 1$  worlds from the final state. (The existence of stable such vacua, however, is suggested in string theory [57].) In any case, it is possible that the sum in Eq. (52) consists of only states in certain limited vacua, although the states themselves may contain complex objects, such as a fractal of stable black holes filling fixed portions of the sky [58]. A detailed study of this issue is warranted, especially in the context of full string theory.

## 8.2 The multiverse as a transient phenomenon

What is the “beginning” of the multiverse? From the observed second law of thermodynamics (in our “vicinity” in the multiverse), we expect that the initial state of the multiverse is a (extremely) low entropy state. (In fact, this is almost the only way to understand the arrow of time from the statistical point of view.) Let  $|\alpha_{\text{beginning}}\rangle$  be such a state. (We treat it as a pure state, for

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<sup>36</sup>Unbroken gauge symmetries do not cause a problem, since at any given time, components of  $|\Psi(t)\rangle$  have vanishing gauge charges because of the charge neutrality.

<sup>37</sup>There is a (small) possibility that an anti de Sitter vacuum transitions into a de Sitter vacuum due to a bubble collision [54]. This, however, does not affect our conclusion.

<sup>38</sup>With a (presumably huge) number of anti de Sitter vacua, non-supersymmetric Minkowski vacua will not be absolutely stable, even taking into account gravitational effects [55]. It is, however, a logical possibility that there are stable such vacua, in which case they should be added to the right-hand side of Eq. (52). Such an addition would not affect discussions below.

simplicity, but an extension to the mixed state case is straightforward.) What do we know about the evolution of the multiverse afterwards?

Consider a complete set of states  $|\alpha_a\rangle$  defined at infinite Minkowski future, namely all possible past light cones of future time-like infinity of Minkowski spacetime, available in the landscape. In view of the result in the previous subsection, the evolution of the multiverse is given by

$$|\alpha_{\text{beginning}}\rangle \rightarrow |\Psi(t = +\infty)\rangle = \sum_a c_a |\alpha_a\rangle. \quad (53)$$

Here, the coefficients  $c_a$  are given by  $c_a = \langle \alpha_a | e^{-i\hat{H}t} | \alpha_{\text{beginning}} \rangle$  with  $t \rightarrow \infty$ , where  $\hat{H}$  is the “time evolution operator” of full quantum gravity in our parametrization of spacetime.

An interesting feature of Eq. (53) is that the final state is given at infinite Minkowski future, where the basis of local operators is particularly simple, and asymptotically free particles are well defined. (In fact, it is these properties that allow us to take the limit  $t \rightarrow +\infty$  without difficulty.) This might be helpful when we try to calculate  $c_a$  in quantum gravity, e.g., by summing up all possible “string world sheets” for a fixed  $|\alpha_a\rangle$ . Such a calculation, however, is far beyond the current technology; it requires, at least, incorporation of all the (including nonperturbative) effects in time-dependent (not necessarily asymptotically Minkowski or anti de Sitter) geometries. Therefore, we focus here on what we can say about Eq. (53) without having explicit information from quantum gravity.

Is there anything we can say about  $|\Psi(t = +\infty)\rangle$  without input from quantum gravity? We expect it to have a fractal structure when expanded in the basis  $|\alpha_a\rangle$ , determined by local operators. We first note that unitarity of the evolution implies that there are nonzero probabilities of forming  $|\alpha_{\text{beginning}}\rangle$  (whatever it is) from other states. For example, if  $|\alpha_{\text{beginning}}\rangle$  is a vacuum with a Planckian energy density, it will be formed in lower energy de Sitter vacua through upward transitions. This implies that  $|\alpha_{\text{beginning}}\rangle$  will appear *as a component* in  $|\Psi(t)\rangle$  at some (late) time  $t_I$ , which will start the entire multiverse as a branch of  $|\Psi(t)\rangle$  for  $t > t_I$ . In fact, this process will be repeated an infinite number of times, making  $|\Psi(t = +\infty)\rangle$  fractal; more precisely,  $c_a(t_\Lambda)$  in  $|\Psi(t_\Lambda)\rangle$  have a fractal structure as  $t_\Lambda \rightarrow \infty$ . Note that this does not affect the well-definedness of the probabilities, since the occurrence of  $|\alpha_{\text{beginning}}\rangle$  in  $|\Psi(t)\rangle$  is exponentially suppressed, because of the difference of the density of states between  $|\alpha_{\text{beginning}}\rangle$  and vacua in which  $|\alpha_{\text{beginning}}\rangle$  is generated.

The picture presented here indicates that our entire multiverse is a transient phenomenon while a low entropy state relaxes into a supersymmetric Minkowski state that has a fractal structure. A natural question is: what is the origin of  $|\alpha_{\text{beginning}}\rangle$ ? As discussed in Section 5.2, if  $|\alpha_{\text{beginning}}\rangle$  is *the* real initial state, then the quantum observer principle is violated there since we cannot evolve the state further back. Is there any way of avoiding this conclusion? In Section 8.4 we will study this issue, and speculate that  $|\alpha_{\text{beginning}}\rangle$  may not be the real beginning, and that the quantum

observer principle may be respected throughout the entire history of spacetime.

### 8.3 General covariance, the arrow of time, and a holographic quantum multiverse

A general covariant theory of gravity based on the global spacetime picture has huge redundancies in its description of physics. There are (at least) three kinds of redundancies:

- **General covariance** — The theory is formulated in such a way that the form of physical laws is invariant under arbitrary coordinate transformations. This introduces large redundancies. While quantities appearing in the theory may depend on (arbitrarily chosen) coordinates, only the ones that are invariant under coordinate transformations are physically observable.
- **Global spacetime** — The theory describes (global) spacetime in such a way that some portions of it are redundant. This occurs when there are spacetime regions that cannot have causal contact later. A classic example of this is given by a black hole [7]: having both the interior of the horizon *and* Hawking radiation within a single description is double counting.
- **Local spacetime** — While a naive picture of spacetime suggests that an  $O(1)$  amount of information can be stored per each Planck size region, the actual number of physical degrees of freedom is much smaller [8]. In fact, the maximal number of degrees of freedom is bounded by the area of the “holographic screen” in Planck units [26].

The first of these three already appears at the level of classical general relativity, while the last two at the quantum (semi-classical) level. It is important to treat these redundancies appropriately when we apply quantum mechanics to a system with gravity.

Our framework explicitly addresses the first two. Recall that we describe a system from the viewpoint of a single observer, parametrizing spacetime *using (past) light cones*. Since causal relations between events are invariant under coordinate transformations, this extracts invariant information about the system. Of course, our time parametrization (in terms of the proper time along the observer) should also be a redundant gauge choice. Indeed, there is no concept of absolute time in quantum gravity—“time evolution” we perceive is simply correlations between physical quantities [10]. Our framework incorporates this idea by formulating physical questions in the form of Eqs. (28, 29) (or Eqs. (9, 10)), in which time  $t$  is nothing but an auxiliary parameter relating different physical events or configurations. For example, when we describe a location of a ball as a function of time,  $\mathbf{x}(\tau)$ , what we really mean is the value of  $\mathbf{x}$  that provides a nonzero support of

$$P(\text{The ball is at } \mathbf{x} \mid \text{The hands of a clock show } \tau \text{ “in our universe”}). \quad (54)$$

for each  $\tau$ , where the phrase “in our universe” represents the conditions needed to specify “a clock” in the multiverse, e.g. the model of a clock, the configuration of other neighboring objects (including ourselves), the fact that the entire system is located on a planet called the earth, which is in a universe whose low energy effective theory is given by an  $SU(3)_C \times SU(2)_L \times U(1)_Y$  gauge theory, etc. etc.<sup>39</sup> The rest of general covariance can also be satisfied straightforwardly by making the “time evolution” operator  $\hat{H}$ , as well as projection operators  $\mathcal{O}_A$ , invariant under “spatial” coordinatizations. The redundancy of global spacetime (the second item in the above list) does not exist either, since we limit our description to the regions that can be physically observed.

The resulting description is, as we discussed throughout this paper, built on the Hilbert space for bulk and horizon degrees of freedom, Eq. (21), as well as the “time evolution” operator  $\hat{H}$  encoding dynamics of full quantum gravity. What form does  $\hat{H}$  take? We have not discussed it explicitly, but it is likely to be rather complicated. Indeed, even in standard QED, the gauge-fixed Hamiltonian (in Coulomb gauge) contains an apparent, Lorentz-violating instantaneous force, whose effects are canceled only after performing full quantum calculations [59]. The situation in a gravitational theory is expected to be worse, especially because the holographic principle implies that the number of degrees of freedom in the bulk is (much) smaller than that indicated by local quantum field theory (as described as the third class of redundancies in the list).

It is possible, however, that a description of the bulk exists in which all the redundancies are fixed, and that such a “holographic description” has a simple(r) form of “time evolution” operator. In this respect, it is encouraging that such simple holographic descriptions do seem to exist (at least) in certain limited cases [60, 61]. Since the holographic principle indicates the existence of a holographic description at the horizon  $\partial\mathcal{M}$  for each semi-classical spacetime  $\mathcal{M}$ , the Hilbert space for the holographic description of the multiverse would take the form

$$\mathcal{H} = \oplus_{\mathcal{M}} (\tilde{\mathcal{H}}_{\mathcal{M},\text{bulk}} \otimes \mathcal{H}_{\mathcal{M},\text{horizon}}). \quad (55)$$

Here,  $\tilde{\mathcal{H}}_{\mathcal{M},\text{bulk}}$  represents the holographic bulk Hilbert space, whose size is manifestly

$$\ln(\dim \tilde{\mathcal{H}}_{\mathcal{M},\text{bulk}}) = \frac{\mathcal{A}_{\partial\mathcal{M}}}{4l_P^2}, \quad (56)$$

where  $\mathcal{A}_{\partial\mathcal{M}}$  is the area of the horizon  $\partial\mathcal{M}$ . Since the dimension of  $\mathcal{H}_{\mathcal{M},\text{horizon}}$  is expected to be similar to that of  $\tilde{\mathcal{H}}_{\mathcal{M},\text{bulk}}$ , it is possible that these two subspaces are actually the same, having a one-to-one correspondence between the elements (as discussed in Section 4.7). If this is the case, then the complete Hilbert space is actually the square root of Eq. (55).

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<sup>39</sup>Here, we have assumed, for simplicity, that  $P(\mathbf{x}|\tau)$  is nonzero only at some value of  $\mathbf{x}$  for a fixed  $\tau$ ; otherwise,  $\mathbf{x}(\tau)$  should be given by a certain average, e.g.,  $\int \mathbf{x} P(\mathbf{x}|\tau) d\mathbf{x} / \int P(\mathbf{x}|\tau) d\mathbf{x}$ .

The evolution of the multiverse state starts from some initial “point” in  $\mathcal{H}$ ,  $|\alpha_{\text{beginning}}\rangle$ , and ends up with a point corresponding to a supersymmetric Minkowski world.<sup>40</sup> One may consider a course-grained entropy of this system defined by Eq. (56), i.e.  $S \equiv \ln(\dim \tilde{\mathcal{H}}_{\mathcal{M},\text{bulk}})$ . The evolution of the multiverse is then

$$S_{\text{beginning}} \sim 0 \quad \rightarrow \quad S_{\text{Minkowski}} = \infty, \quad (57)$$

where  $S_{\text{beginning}} \sim 0$  simply means that the entropy of  $|\alpha_{\text{beginning}}\rangle$  is low, e.g. somewhere in the range between 0 and a few orders of magnitude. This evolution of  $S$  is ultimately the origin of the “global arrow of time” in the multiverse, which also dictates our local arrow of time.

## 8.4 The fractal “mega-multiverse”

What is the origin of  $|\alpha_{\text{beginning}}\rangle$  at the beginning of our multiverse? Does it arise somehow from outside our framework? Or can we understand it consistently within the framework?

Recall that no matter what  $|\alpha_{\text{beginning}}\rangle$  is, it also appears at late times as components in  $|\Psi(t)\rangle$ , reproducing the entire multiverse from there. Then, why can’t we identify “the initial state” of  $|\Psi(t)\rangle$ , from which we started, to arise as a component in a “larger” state  $|\Phi(t)\rangle$ ? In some sense, this is unavoidable. Since the same  $|\alpha_{\text{beginning}}\rangle$  appears multiple (infinitely many) times during the evolution, we will not know which branch “our multiverse” corresponds to. In this picture, the initial state of our multiverse arises as a statistical fluctuation in a larger structure  $|\Phi(t)\rangle$ . It implies that we only know the initial state statistically, i.e., our multiverse begins as a mixed state

$$\rho_{\text{beginning}} \simeq \sum_i \lambda_i |\alpha_{\text{beginning},i}\rangle \langle \alpha_{\text{beginning},i}|, \quad (58)$$

where  $\lambda_i$  is the probability of producing  $|\alpha_{\text{beginning},i}\rangle$  as a component in  $|\Phi(t)\rangle$ . Grouping together the states having the same semi-classical geometry  $a$ , Eq. (58) can be written as

$$\rho_{\text{beginning}} \simeq \sum_a \lambda_a \rho_{\text{beginning},a}, \quad \rho_{\text{beginning},a} \equiv \frac{1}{\sum_{i \in a} \lambda_i} \sum_{i \in a} \lambda_i |\alpha_{\text{beginning},i}\rangle \langle \alpha_{\text{beginning},i}|. \quad (59)$$

What about the beginning of  $|\Phi(t)\rangle$ ? Given a landscape potential, we can explore dynamics of a structure larger than our multiverse, defined as a particular branch evolving as Eq. (57). The relevant machinery was developed in Ref. [63] in the context of geometric cutoff measures. Let us introduce an object that captures coarse-grained dynamics of the large(st) structure at the semi-classical level:

$$\aleph(t) = \sum_a f_a(t) \rho_a, \quad (60)$$

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<sup>40</sup>Our picture shares some features in common with the proposal of Refs. [61, 62], which claims that the entire multiverse is described by a two-dimensional Euclidean Liouville theory on the boundary of a stable Minkowski bubble. A relation between the two pictures, if any, is not clear.

where  $\rho_a$  is the density matrix representing the states in vacuum  $a$ . The rate equation governing the evolution of  $\mathfrak{N}(t)$  is given by

$$\frac{df_a}{dt}(t) = M_{ab} f_b(t), \quad (61)$$

where  $M_{ab}$  is the transition matrix between different vacua.

The solutions of Eq. (61) can be written in terms of (generalized) eigenvectors  $v_a^X$  of  $M_{ab}$ , which form a complete basis, i.e.,  $X$  runs from 1 to the number of vacua  $n$ . The eigenvalues and eigenvectors of  $M_{ab}$  have the following structure [63]. Each terminal vacuum is an eigenvector with eigenvalue zero, which we denote by  $v_a^I$  ( $I = 1, \dots, n_T$ ), where  $n_T$  is the number of terminal vacua. All the other eigenvalues have negative real parts. In particular, the eigenvalue that has the smallest real part in magnitude,  $\alpha_D$ , is pure real and has a nondegenerate eigenvector  $v_a^D$  (called the dominant eigenvalue/eigenvector). The rest of the eigenvalues  $\alpha_L$  may or may not be degenerate, having (generalized) eigenvectors  $v_a^L$ , where  $L = 1, \dots, n_S$  with  $n_S + n_T + 1 = n$ . The general solution to Eq. (61) is then given by

$$f_a(t) = \sum_{I=1}^{n_T} c_I v_a^I + c_D e^{-|\alpha_D|t} v_a^D + \sum_{L=1}^{n_S} c_L(t) e^{-|\text{Re } \alpha_L|t} v_a^L, \quad (62)$$

where  $c_{I,D}$  are constants, and  $c_L(t)$  are composed of polynomial and trigonometric functions.

There are two scenarios one can imagine here:

**The multiverse with a beginning** — There is some theory beyond our framework, e.g. creation from “nothing” or quantum gravity, that provides the initial condition for the *largest possible* structure. In this case, we can simply call this structure the multiverse, and the situation is reduced to that discussed in Section 5.2. The quantum observer principle is violated at the earliest moment, since we cannot evolve the state further back. In general, the phenomenological predictions do depend on the initial condition.

**The stationary, fractal “mega-multiverse”** — Instead of admitting the existence of the “beginning,” we may require that the quantum observer principle is respected for the *whole history* of spacetime. In this case, the beginning of our multiverse is a fluctuation of a larger structure, whose beginning is also a fluctuation of an even larger structure, and this series goes on forever. This leads to the picture that our multiverse arises as a fluctuation in a huge, *stationary* “mega-multiverse,” which has a fractal structure. Here the “stationary” means that the same predictions are obtained by Eq. (28) (or Eq. (29)) regardless of the limits of the  $t$  integrals, as long as the interval is taken sufficiently long. For this to be true, inflating regions of the mega-multiverse state must be in one of the eigenvectors of  $M_{ab}$ . Assuming that we are interested only in “transient phenomena” in the multiverse, the initial condition *for our multiverse* can be taken to be

$$\lambda_a \propto \lambda_{ab} H_b^{-3} v_b^D \quad \text{or} \quad \lambda_{ab} H_b^{-3} v_b^L, \quad (63)$$

where  $\lambda_{ab}$  is the rate of the transition  $b \rightarrow a$  per unit physical spacetime,  $H_b$  the Hubble parameter in vacuum  $b$ , and  $L$  can be any of the  $n_S$  possibilities. In this scenario, no “real” initial condition needs to be imposed—there is simply no beginning or end for the mega-multiverse. On the other hand, we need to choose which of the stationary states the mega-multiverse is in. This is, in some sense, a choice of “theories” (rather than “vacua” or “initial conditions”) because there is no physical process allowing transitions between different choices.

In either of the scenarios described above, our multiverse is one of the infinite series of multiverses created in a larger structure as statistical fluctuations. The global arrow of time in Eq. (57)—a part of which is our arrow of time—is simply a manifestation of the fluctuations relaxing into the equilibrium state of a supersymmetric Minkowski fractal.

Can we explore the larger structure  $|\Phi(t)\rangle$  experimentally by evolving our current multiverse state back in time?<sup>41</sup> Unfortunately, we cannot. If we evolve the multiverse state to the past, we would at some point reach one of the “initial states,”  $|\alpha_{\text{beginning},i}\rangle$ . To go back further, however, we need to know other components in  $|\Phi(t)\rangle$ , which are “outside” our multiverse. In the conventional language, our multiverse state  $|\Psi(t)\rangle$  is a state after “wavefunction collapse,” i.e. a state obtained after throwing away components that are irrelevant for *future* measurements (namely future of  $|\alpha_{\text{beginning},i}\rangle$ ). But to evolve the state further back, it is not enough to know the wavefunction “already collapsed”; we need to know the state “before the collapse.” This may make one worry that the considerations here might just be “meta-physics.” However, the scenarios described here *do* affect our multiverse through their implications on the initial condition, e.g. through Eq. (63), whose consequences can, in principle, be worked out and compared with the observations. At the very least, different choices of the scenario lead to different fractal patterns in the final Minkowski bubbles, which can be observed by civilizations (if any) living in these bubbles. Since the coarse-grained entropy of Minkowski space is infinite, there is enough room in these bubbles to store all the information about the “beginning.”

The scenarios presented here are speculative, but attractive. In particular, the fractal mega-multiverse finally eliminates the necessity of imposing initial conditions from cosmology (though at the cost of introducing a choice of the state), and it seems to be an inevitable consequence of the ultimate extrapolation of the quantum observer principle. Maybe, quantum mechanics already tells us the entire history of the multiverse, and even about an infinite series of multiverses, which are created in stationary, fractal spacetime.

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<sup>41</sup>Note that, because of the deterministic nature of quantum evolution, we do not need to regard that time flows from smaller to larger  $t$ —we could equally view that time evolves the other way. Such a picture is obviously highly unintuitive—for example, our brains “evolve in time” in such a way that we keep losing our “memories of the future”—but this is not a real problem. The real problem is that such a description is highly sensitive to small perturbations; namely, the evolution of the system is unstable against small errors in the initial data. Here, we assume that we have a perfect knowledge about the current state of the multiverse.

## Acknowledgments

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## A Interpretation as a “Fuzzy” Time Cutoff

In expanding universes, our method in Section 3.1 can be viewed as a sort of “fuzzy” time cutoff. Suppose that prior conditions  $A$  can be satisfied in a class of vacua  $X$  if they are nucleated inside any of vacua  $Y$ . We assume that the conditions are met in a fraction  $r_X$  of the spatial volume at time  $\tau_{X,\text{obs}}$  after the nucleation, where  $\tau_X$  is the Friedmann-Robertson-Walker time inside  $X$  bubbles. Now, the comoving volume of a bubble  $X$  nucleated in  $Y$  is

$$V_{Y \rightarrow X} = \frac{4\pi}{3} H_Y^{-3} e^{-3t_{\text{nuc}}}, \quad (64)$$

where  $t_{\text{nuc}}$  is the scale factor time at the bubble nucleation, which depends on  $X$  and  $Y$ , and  $H_Y$  is the Hubble expansion rate in vacuum  $Y$ .

Now, consider a set of geodesics each occupying comoving volume  $V_\epsilon = \epsilon^3$ . On average, a past light cone satisfying  $A$  intersects with one of the geodesics if  $r_X V_{Y \rightarrow X} \gtrsim V_\epsilon$ , i.e.

$$t_{\text{nuc}} \lesssim \ln \frac{(\frac{4\pi}{3} r_X)^{1/3}}{\epsilon H_Y}, \quad (65)$$

where we have adopted the square bubble approximation, which is appropriate in the present context [64]. This implies that light cones are “counted” only if bubbles to which they belong form early enough, and therefore provides an effective time cutoff which becomes infinity for  $\epsilon \rightarrow 0$ . This cutoff, however, is “fuzzy” and depends on properties of  $X$  and  $Y$ .

Incidentally, the well-definedness of quantum probabilities, introduced in Section 4, can also be understood similarly. While a particular event may happen multiple times in the history of the multiverse, the probabilities of that to occur decrease exponentially with the number of times. Thus, events that happen later make smaller contributions in calculation of the probabilities (e.g. to the numerators and denominators of Eqs. (28) and (29)). This provides an effective time cutoff in the quantum probabilities.

## B Sample Calculations in Toy Landscapes

### B.1 Semi-classical picture

Here we present explicit calculations of probabilities in our framework, using simplified toy landscape models. We assume that all the vacua lead to physical laws that reduce to the standard model of particle physics at low energies; namely, they differ only in properties that are not yet measured experimentally, e.g. the Higgs boson mass or TeV-scale physics, which we assume not to affect our own evolution in these vacua. We also suppose that certain transitions between vacua lead to the standard model of cosmology; for example, they provide sufficiently high reheating temperature that later histories are consistent with the current observations. We see that under these simplifying assumptions, the calculations reduce essentially to those for comoving probabilities [65].

Let us consider the landscape consisting of several discrete vacua. The fraction of comoving volume occupied by vacuum  $X$  at time  $t$ ,  $f_X(t)$ , then obeys the following rate equation:

$$\frac{df_X}{dt} = \sum_Y \left( \kappa_{XY} - \delta_{XY} \sum_Z \kappa_{ZX} \right) f_Y, \quad (66)$$

where  $\kappa_{XY} \equiv (4\pi/3)\lambda_{XY}H_Y^{-4}$ , and  $\lambda_{XY}$  and  $H_Y$  are the bubble nucleation rate per unit physical spacetime for the  $Y \rightarrow X$  transition and the Hubble expansion rate in vacuum  $Y$ , respectively. Here, we have taken  $t$  to be the scale factor time, i.e.  $t = \ln a(t)$  where  $a(t)$  is the scale factor, but the final results do not depend on the choice of the time coordinate.

Now, we want to find the past light cones that are consistent with our prior conditions and encountered by geodesics emanating from an initial space-like hypersurface  $\Sigma$ . Assuming that transitions  $Y \rightarrow X$  lead to standard cosmology, the number of such past light cones whose tips are in vacuum  $X$  can be estimated as

$$\mathcal{N}_X \propto \sum_Y \int_0^{t_c} \frac{V_{XY}(t)}{V_\epsilon} dt \propto \sum_Y \int_0^{t_c} \kappa_{XY} f_Y(t) dt, \quad t_c \approx \ln \left[ \frac{(4\pi/3)^{1/3}}{\epsilon H_Y} \right], \quad (67)$$

where  $V_{XY}(t) dt$  and  $V_\epsilon = \epsilon^3$  are the comoving volume of  $X$  created by the transition  $Y \rightarrow X$  between time  $t$  and  $t + dt$  and the average comoving volume occupied by a single geodesic, respectively. The function  $f_Y$  is obtained by solving Eq. (66) with a given initial condition  $f_X(0)$ , and the “cutoff time”  $t_c$  is determined from the fact that the comoving volume of a bubble formed after  $t_c$  is smaller than  $\epsilon^3$ , so that the geodesics typically do not intersect with these bubbles. The relative probability of finding ourselves in vacua  $X_1$  and  $X_2$  is then given by

$$\frac{P_{X_1}}{P_{X_2}} = \lim_{\epsilon \rightarrow 0} \frac{\mathcal{N}_{X_1}}{\mathcal{N}_{X_2}}. \quad (68)$$

This probability agrees with the comoving probability.

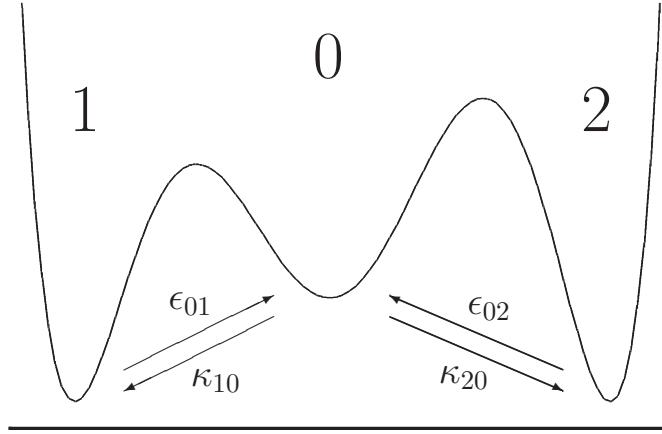


Figure 11: Toy landscape (I)—A system with three recyclable vacua.

The calculation can be simplified significantly if the landscape possesses (at least one) terminal vacua, as suggested by string theory. In this case, we can use the integrated version of Eq. (66):

$$f_X(\infty) - f_X(0) = \sum_Y \left( \kappa_{XY} - \delta_{XY} \sum_Z \kappa_{ZX} \right) F_X, \quad (69)$$

where  $F_X \equiv \int_0^\infty f(t) dt$ . Since  $f_X(\infty) = 0$  for *non-terminal* vacua (as all the comoving volume decays into terminal vacua at  $t \rightarrow \infty$ ), we can obtain  $F_X$  for these vacua by simply solving the set of linear equations (69) for a given initial condition  $f_X(0)$ . The probabilities are then obtained using Eq. (67), which now takes the form

$$\lim_{\epsilon \rightarrow 0} \mathcal{N}_X \propto \sum_Y \kappa_{XY} F_Y. \quad (70)$$

Note that the summation in the right-hand side runs only over non-terminal vacua, so that we need to solve Eq. (69) only for these vacua.

We now apply the above formulae to some simple examples.

### Example 1—A system with three recyclable vacua

Let us consider a system with three de Sitter vacua between which there are several allowed transitions as depicted in Fig. 11. The rate equation (66) is then given by

$$\frac{d}{dt} \begin{pmatrix} f_0 \\ f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} -(\kappa_{10} + \kappa_{20}) & \epsilon_{01} & \epsilon_{02} \\ \kappa_{10} & -\epsilon_{01} & 0 \\ \kappa_{20} & 0 & -\epsilon_{02} \end{pmatrix} \begin{pmatrix} f_0 \\ f_1 \\ f_2 \end{pmatrix}, \quad (71)$$

where we have written the rates for upward transitions (transitions from lower energy minima to higher energy ones) as  $\epsilon_{XY}$ , instead of  $\kappa_{XY}$ , to emphasize that they are generically much smaller than the rates for usual, downward transitions [66]. We assume that the standard cosmology is obtained when the universe experiences a transition  $0 \rightarrow 1$  or  $0 \rightarrow 2$ , so that we can be living either vacuum 1 or 2.<sup>42</sup>

Suppose the initial condition of spacetime is given by

$$f_0(0) = 1, \quad f_1(0) = f_2(0) = 0, \quad (72)$$

i.e. the multiverse begins from the highest energy minimum. Then, Eq. (71) gives

$$f_0(t) = e^{-(\kappa_{10} + \kappa_{20})t} + \frac{\epsilon_{01}\kappa_{10} + \epsilon_{02}\kappa_{20}}{(\kappa_{10} + \kappa_{20})^2} \{1 - e^{-(\kappa_{10} + \kappa_{20})t} - t(\kappa_{10} + \kappa_{20})e^{-(\kappa_{10} + \kappa_{20})t}\} + O(\epsilon^2), \quad (73)$$

$$f_1(t) = \frac{\kappa_{10}}{\kappa_{10} + \kappa_{20}} \{1 - e^{-(\kappa_{10} + \kappa_{20})t}\} + O(\epsilon), \quad (74)$$

$$f_2(t) = \frac{\kappa_{20}}{\kappa_{10} + \kappa_{20}} \{1 - e^{-(\kappa_{10} + \kappa_{20})t}\} + O(\epsilon), \quad (75)$$

so that  $\int_0^T f_0(t) dt = ((\epsilon_{01}\kappa_{10} + \epsilon_{02}\kappa_{20})/(\kappa_{10} + \kappa_{20})^2)T$  for  $T \rightarrow \infty$ . The relative probability for finding ourselves in vacua 1 and 2 is thus

$$\frac{P_1}{P_2} = \lim_{T \rightarrow \infty} \frac{\int_0^T \kappa_{10} f_0(t) dt}{\int_0^T \kappa_{20} f_0(t) dt} = \frac{\kappa_{10}}{\kappa_{20}}. \quad (76)$$

Similarly, if the initial condition is given by

$$f_1(0) = 1, \quad f_0(0) = f_2(0) = 0, \quad (77)$$

then

$$f_0(t) = \frac{\epsilon_{01}}{\kappa_{10} + \kappa_{20}} \{1 - e^{-(\kappa_{10} + \kappa_{20})t}\} + O(\epsilon^2), \quad (78)$$

$$f_1(t) = 1 + O(\epsilon), \quad (79)$$

$$f_2(t) = O(\epsilon), \quad (80)$$

leading to  $\lim_{T \rightarrow \infty} \int_0^T f_0(t) dt = (\epsilon_{01}/(\kappa_{10} + \kappa_{20}))T$ . The probability is, thus, again given by

$$\frac{P_1}{P_2} = \frac{\kappa_{10}}{\kappa_{20}}. \quad (81)$$

We find that the relative probability does not depend on the initial condition in this particular example. This can be understood from the fact that “our universe” arises only through a transition from vacuum 0 to 1 or 2, and that the branching ratio for these transitions is given by  $\text{Br}(0 \rightarrow 1)/\text{Br}(0 \rightarrow 2) = \kappa_{10}/\kappa_{20}$ .

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<sup>42</sup>A landscape without a terminal vacuum will be unrealistic, as it leads to the problem of Boltzmann brains; see Section 7.1. Here we consider such a model simply for illustrative purposes.

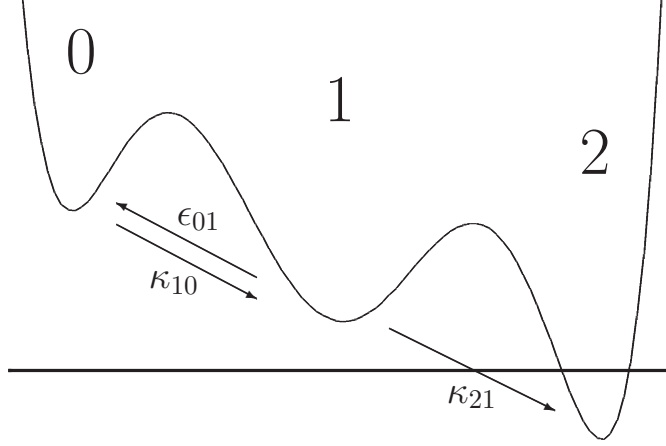


Figure 12: Toy landscape (II)—A system with two non-terminal and one terminal vacua.

### Example 2—A system with two non-terminal and one terminal vacua

We now consider a system with two non-terminal and one terminal vacua, illustrated in Fig. 12. The rate equation for this system is given by

$$\frac{d}{dt} \begin{pmatrix} f_0 \\ f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} -\kappa_{10} & \epsilon_{01} & 0 \\ \kappa_{10} & -(\kappa_{21} + \epsilon_{01}) & 0 \\ 0 & \kappa_{21} & 0 \end{pmatrix} \begin{pmatrix} f_0 \\ f_1 \\ f_2 \end{pmatrix}. \quad (82)$$

As discussed before, we can focus on the integrated version of this equation for non-terminal vacua:

$$- \begin{pmatrix} f_0(0) \\ f_1(0) \end{pmatrix} = \begin{pmatrix} -\kappa_{10} & \epsilon_{01} \\ \kappa_{10} & -(\kappa_{21} + \epsilon_{01}) \end{pmatrix} \begin{pmatrix} F_0 \\ F_1 \end{pmatrix}, \quad (83)$$

where we have used  $f_0(\infty) = f_1(\infty) = 0$ . We assume that the standard cosmology arises after the transition  $0 \rightarrow 1$  or  $1 \rightarrow 2$ .<sup>43</sup>

If the initial condition is given by

$$f_0(0) = 1, \quad f_1(0) = 0, \quad (84)$$

i.e. the multiverse starts from the higher de Sitter minimum, then Eq. (83) gives

$$F_0 = \frac{\kappa_{21} + \epsilon_{01}}{\kappa_{10}\kappa_{21}}, \quad F_1 = \frac{1}{\kappa_{21}}, \quad (85)$$

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<sup>43</sup>Here we ignore the fact that we have already measured a positive vacuum energy in our universe.

and the relative probability of finding ourselves in vacua 1 and 2 is

$$\frac{P_1}{P_2} = \frac{\kappa_{10}F_0}{\kappa_{21}F_1} = \frac{\kappa_{21} + \epsilon_{01}}{\kappa_{21}}. \quad (86)$$

On the other hand, if the multiverse starts from the lower de Sitter minimum

$$f_0(0) = 0, \quad f_1(0) = 1, \quad (87)$$

then

$$F_0 = \frac{\epsilon_{01}}{\kappa_{10}\kappa_{21}}, \quad F_1 = \frac{1}{\kappa_{21}}, \quad (88)$$

so that

$$\frac{P_1}{P_2} = \frac{\epsilon_{01}}{\kappa_{21}}. \quad (89)$$

The results in Eqs. (86) and (89) show that the probability does depend on the initial condition. The physical picture in each case is clear. Let us first consider the former case, where the initial condition is given by Eq. (84). In this case, if the multiverse starting from vacuum 0 simply decays into 2, i.e. if  $\epsilon_{01} = 0$ , then we would obtain  $P_1/P_2 = 1$  since the decay must always occur through vacuum 1. Now, turning on a recycling process  $\epsilon_{01} \neq 0$  makes  $P_1/P_2$  slightly larger than 1, since there is then a small possibility of the universe experiencing the transition  $0 \rightarrow 1$  more than once, although  $1 \rightarrow 2$  occurs only once. This explains the form of Eq. (86). On the other hand, in the latter case where the initial condition is given by Eq. (87), the multiverse starts from vacuum 1 which, in the limit of  $\epsilon_{01} = 0$ , simply decays into vacuum 2, giving  $P_1/P_2 = 0$ . However, a recycling process provides a small possibility that the transition  $1 \rightarrow 0$  happens before the  $1 \rightarrow 2$  decay, making  $P_1/P_2$  slightly nonzero. In fact, the expression of Eq. (89) is nothing but the branching ratio  $\text{Br}(1 \rightarrow 0)/\text{Br}(1 \rightarrow 2) = \epsilon_{01}/\kappa_{21}$ .

## B.2 Quantum picture

Here we compute probabilities in toy landscapes, using the quantum mechanical definition given in Section 4. We consider the same setup as in Section B.1: all the vacua lead to the standard model of particle physics at low energies, and measurements are performed (i.e. civilizations exist) right after certain cosmic phase transitions. Note that while we adopt the quantum mechanical definition of the probabilities, the computation is (necessarily) semi-classical, as we do not know the theory of quantum gravity.

Following Section 4.4, we describe the multiverse in terms of bulk density matrices. With the level of approximation we need, the complete set for these density matrices  $\mathcal{S}$  can be taken as all possible past light cones whose tips are in vacuum  $X$  at proper time  $\tau$  after the last bubble nucleation:

$$\mathcal{S} = \{\rho_{X,\tau}\}. \quad (90)$$

The general multiverse state can then be written as

$$\rho_{\text{bulk}}(t) = \sum_X \int d\tau C_{X,\tau}(t) \rho_{X,\tau}, \quad (91)$$

where  $t$  represents proper time along the observer (geodesic). The evolution of the coefficients  $C_{X,\tau}(t)$  can be calculated semi-classically, and is governed by the usual rate equations:

$$|C_{X,0}(t + \Delta t)|^2 = \sum_Y \lambda_{XY} \frac{4\pi}{3H_Y^3} \int d\tau |C_{Y,\tau}(t)|^2, \quad (92)$$

$$|C_{X,\tau+\Delta\tau}(t + \Delta t)|^2 = |C_{X,\tau}(t)|^2 - \sum_Z \lambda_{ZX} \frac{4\pi}{3H_X^3} \Delta\tau |C_{X,\tau}(t)|^2, \quad (93)$$

where  $\Delta\tau = \Delta t$  by construction, and  $\lambda_{XY}$  and  $H_Y$  are as defined in Section B.1. Defining

$$f_X(t) = \int d\tau |C_{X,\tau}(t)|^2, \quad (94)$$

i.e. the probability of the tip of the light cone being in vacuum  $X$  at time  $t$ , the evolution equation for  $f_X$  is obtained from Eqs. (92) and (93) as

$$\frac{df_X(t)}{dt} = \sum_Y \lambda_{XY} \frac{4\pi}{3H_Y^3} f_Y(t) - \sum_Z \lambda_{ZX} \frac{4\pi}{3H_X^3} f_X(t). \quad (95)$$

Now, suppose that the transitions to vacua  $X_1$  and  $X_2$  lead to intelligent life just after the transitions. An important point is that the anthropic factor  $n_X$ , i.e. the probability of finding an experimenter in vacuum  $X$ , is the same for *all bubbles* with the *same vacuum*  $X$ , because all these bubbles look identical to the observer (geodesic) traveling the multiverse. In particular, bubbles formed at later times are *not* rewarded by the volume increase during eternal inflation. Assuming that  $X_1$  and  $X_2$  have equal anthropic factors (as in Section B.1 and also implicit in Eq. (91)), the relative probability of finding these vacua is given through the definition of Eq. (34) as

$$\frac{P_{X_1}}{P_{X_2}} = \frac{\int dt \text{Tr}\{\rho_{\text{bulk}}(t) \mathcal{O}_{\text{bulk},X_1}\}}{\int dt \text{Tr}\{\rho_{\text{bulk}}(t) \mathcal{O}_{\text{bulk},X_2}\}} = \frac{\int dt |C_{X_1,0}(t)|^2}{\int dt |C_{X_2,0}(t)|^2}, \quad (96)$$

where  $\mathcal{O}_{\text{bulk},X_i} = \rho_{X_i,0}$ . The factors appearing in the rightmost expression can be calculated from Eq. (92):

$$\int dt |C_{X,0}(t)|^2 = \sum_Y \lambda_{XY} \frac{4\pi}{3H_Y^3} \int f_Y(t) dt, \quad (97)$$

where  $f_Y(t)$  is obtained as a solution to Eq. (95), once the initial conditions are given.

Incidentally, equations (95) and (97) may be rewritten in terms of scale factor time  $t_{\text{SF}}$ ,  $dt_{\text{SF}} = H dt$ , as

$$\frac{df_X(t_{\text{SF}})}{dt_{\text{SF}}} = \sum_Y \kappa_{XY} f_Y(t_{\text{SF}}) - \sum_Z \kappa_{ZX} f_X(t_{\text{SF}}), \quad (98)$$

$$\int dt |C_{X,0}(t)|^2 = \sum_Y \kappa_{XY} \int f_Y(t_{\text{SF}}) dt_{\text{SF}}, \quad (99)$$

where  $\kappa_{XY} = (4\pi/3)\lambda_{XY}H_Y^{-4}$ , as in Section B.1. We then find, comparing Eqs. (66, 67, 68) and (98, 99, 96), that relative probabilities defined using the quantum picture (in Section 4) precisely agree with those defined using the semi-classical picture (in Section 3).

## C No Quantum Cloning in Bubble Universes

Our framework postulates that all the information behind apparent horizons (as viewed from the observer) is encoded on the (stretched) apparent horizons. For past horizons, this provides “initial conditions” for the subsequent evolution in the bulk of spacetime; and for future horizons, the information stored can be sent back as Hawking radiation. In this Appendix, we provide a nontrivial consistency check of this picture in the eternally inflating multiverse. Our analysis closely follows that of Ref. [67], performed in the context of black hole physics.

Consider an observer in an inflationary phase. We describe the spacetime using the flat coordinates:

$$ds^2 = -dt^2 + \frac{1}{H^2} e^{2Ht} (dr^2 + r^2 d\Omega_2^2), \quad (100)$$

where  $H$  is the Hubble parameter, and we set the observer at  $r = 0$ . Suppose there is a traveler falling behind the observer’s horizon, as depicted in Fig. 13. Then, all the information he/she carries will be stored on the stretched horizon, from the observer’s viewpoint. Let  $t_{\text{esc}}$  be the time when the traveler crosses the horizon. The traveler’s location at  $t = t_{\text{esc}}$  is then given by

$$r_{\text{traveler}} = e^{-Ht_{\text{esc}}}. \quad (101)$$

If the traveler follows a comoving path,  $r_{\text{traveler}}$  stays constant throughout the future history.

According to the present picture, the observer can retrieve the information carried away by the traveler, from Hawking radiation at late times (the wavy arrow in Fig. 13). On the other hand, the traveler may also try to communicate the information to the observer by sending some signal after he/she crosses the horizon (the solid arrow). Now, suppose a Minkowski bubble forms in the future of the observer. Then, the observer may receive the signal sent by the traveler, as shown in Fig. 13. This is a dangerous situation. If the observer could obtain the information *both* from Hawking radiation *and* from the signal, then the observer would have duplicate quantum information, contradicting the basic principles of quantum mechanics: the no-cloning theorem.

Let us examine the condition under which this inconsistency might occur. Suppose, for simplicity, that the Minkowski bubble nucleates at time  $t_{\text{nuc}}$  on the observer’s trajectory,  $r = 0$ . The observer can then collect the information before entering into the bubble, if

$$t_{\text{nuc}} \gtrsim t_{\text{esc}} + t_{\text{ret}}. \quad (102)$$

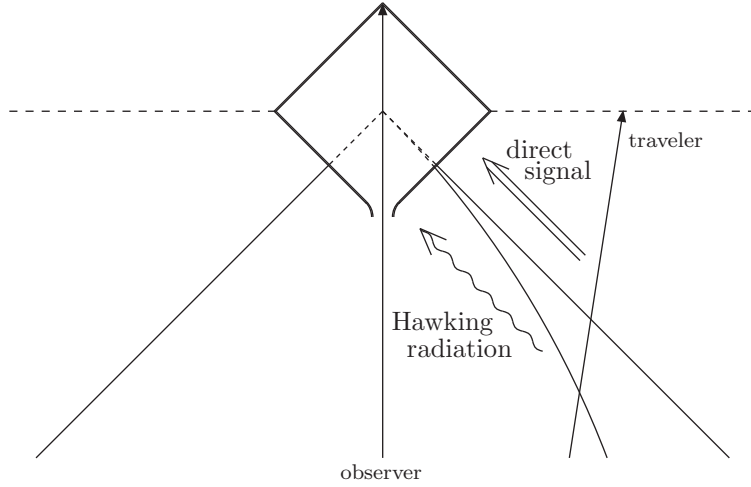


Figure 13: An observer traveling the multiverse may apparently receive the same quantum information both from Hawking radiation and from a direct signal sent by the traveler, which would violate the no-cloning theorem of quantum mechanics. A careful consideration, however, reveals that this cannot happen.

Here,  $t_{\text{ret}}$  is the information retrieval time, i.e. the minimal time needed to retrieve any information from the horizon, which for a de Sitter space is given by [68]

$$t_{\text{ret}} = \frac{1}{H} \left\{ \ln \left( \frac{1}{l_P H} \right) + O(1) \right\}. \quad (103)$$

(Note that the coefficient is fixed to be  $1/2\pi T = 1/H$  where  $T$  is the de Sitter temperature.) In de Sitter space, the bubble nucleated at  $t_{\text{nuc}}$  grows to the size  $r \approx e^{-Ht_{\text{nuc}}}$  at future infinity.<sup>44</sup> Therefore, the largest distance to the bubble wall from the observer who collected the information from Hawking radiation is

$$r_{\text{wall}} \lesssim r_{\text{wall, max}} \approx e^{-Ht_{\text{esc}}} e^{-Ht_{\text{ret}}}. \quad (104)$$

Here, we have used Eq. (102).

We now consider the signal sent by the traveler. Suppose the traveler sent it to the observer at time  $\Delta t$  after he/she crossed the horizon, i.e. at  $t = t_{\text{esc}} + \Delta t$ . The signal then reaches

$$r_{\text{signal}} \gtrsim r_{\text{signal, min}} \approx e^{-Ht_{\text{esc}}} (1 - e^{-H\Delta t}), \quad (105)$$

at future infinity. Here, we have assumed that the traveler follows a comoving trajectory, which is

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<sup>44</sup>Strictly speaking, the metric after the information gathering must change from de Sitter to Schwarzschild-de Sitter, but the correction from this is negligible.

a good approximation for small  $\Delta t$ . The contradiction would occur if

$$r_{\text{signal, min}} < r_{\text{wall, max}}, \quad (106)$$

since then the observer may receive the same information both from Hawking radiation and the signal. Using Eqs. (103), (104), and (105), this is translated into

$$\Delta t \lesssim l_P. \quad (107)$$

The same inequality is also obtained by considering another (extreme) setup where the bubble nucleates on the stretched horizon in the direction antipodal to the traveler.

The above analysis indicates that the contradiction may occur only if the traveler can send the information fast enough (in a super-Planckian time) that the condition of Eq. (107) is satisfied. Is it possible to do that? The holographic principle implies that the amount of information the traveler can emit during time interval  $\Delta t$  is bounded by

$$I \leq \frac{\pi}{4l_P^2} \Delta t^2, \quad (108)$$

so that sending even one bit of information,  $I \approx \ln 2$ , requires the Planck time:

$$\Delta t \gtrsim l_P. \quad (109)$$

Therefore, we find that the violation of the principles of quantum mechanics actually does *not* occur, as viewed from the observer. In fact, it is quite convincing that this “quantum censorship” works but only barely.

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